The interaction of an submerged object with a Very Large Floating Platform

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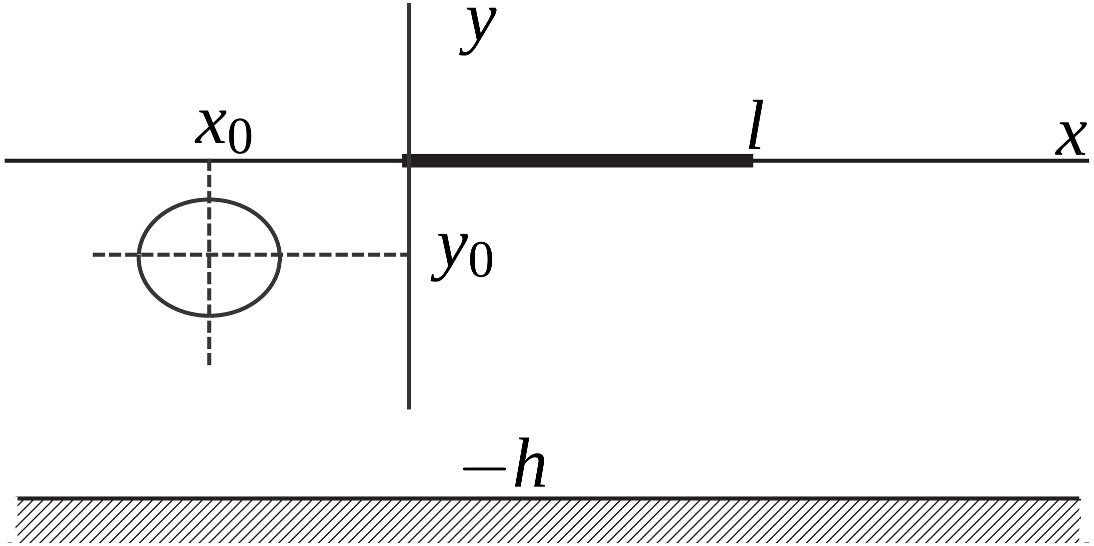

The fluid is ideal, so we introduce the velocity potential $\mathbf{V}(\mathbf{x}, t) = \nabla\Phi(\mathbf{x}, t)$, where $\mathbf{V}(\mathbf{x}, t)$ is the fluid velocity vector. $\Delta\Phi = 0$ in the fluid, together with the linearized kinematic condition, $\Phi_y = \tilde{v}_t$, and a dynamic condition.

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and at the moving platform

$$\left\{\frac{D}{\rho g}\frac{\partial^4}{\partial x^4} + \frac{m}{\rho g}\frac{\partial^2}{\partial t^2} + 1\right\}\frac{\partial\Phi}{\partial y} + \frac{1}{g}\frac{\partial^2\Phi}{\partial t^2} = 0.$$

We assume that the velocity potential is a time-harmonic wave function, $\Phi(\mathbf{x}, t) = \phi(\mathbf{x})\ \mathrm{e}^{-\mathrm{i}\omega t}$. We introduce the following parameters: $K(=\nu) = \frac{\omega^2}{g},\ \mu = \frac{m\omega^2}{\rho g},\ \mathcal{D} = \frac{D}{\rho g}$.

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The potential of the undisturbed incident wave is given by:

$$\phi^{\text{inc}}(\mathbf{x}) = \frac{g\zeta_\infty}{\mathrm{i}\omega}\frac{\cosh(k_0(y+h))}{\cosh(k_0 h)}\exp(\mathrm{i}k_0 x)$$

where ζ_∞ is the wave height in the original coordinate system, ω the frequency, while the wave number k_0 is the positive real solution of the dispersion relation, $k_0 \tanh(k_0 h) = K$, for finite water depth.

If the Green's function obeys the free surface condition we obtain for the total potential:

$$
\begin{aligned}
2\pi\phi(x,y) =& 2\pi\phi^{\text{inc}}(x,y) - \\
&- \int_{\mathcal{C}} \left\{ \phi(\xi,\eta)\frac{\partial \mathcal{G}(x,z;\xi,\eta)}{\partial n} - \frac{\partial \phi(\xi,\eta)}{\partial n}\mathcal{G}(x,z;\xi,\eta) \right\} \, \mathrm{d}s + \\
&+ \int_0^l \left(\phi(\xi,0)\frac{\partial \mathcal{G}(x,y;\xi,0)}{\partial \eta} - \frac{\partial \phi(\xi,0)}{\partial \eta}\mathcal{G}(x,z;\xi,0) \right) \, \mathrm{d}\xi.
\end{aligned}
$$

Depending on the situation we choose a different form of the Green's function

$$\mathcal{G}(x,y;\xi,\eta) = \int_{\mathcal{L}'} \frac{1}{\gamma} \frac{K \sinh \gamma y + \gamma \cosh \gamma y}{K \cosh \gamma h - \gamma \sinh \gamma h} \cosh \gamma(\eta + h)\, \mathrm{e}^{\mathrm{i}\gamma(x-\xi)}\, \mathrm{d}\gamma$$

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$$\text{for } y > \eta$$

$$\mathcal{G}(x,y;\xi,\eta) = -2\pi\mathsf{i} \sum_{i=0}^{\infty} \frac{1}{k_i} \frac{k_i^2 - K^2}{hk_i^2 - hK^2 + K} \cosh k_i(y+h)$$

$$\cosh k_i(\eta + h) \; \mathsf{e}^{\mathsf{i}k_i|x-\xi|},$$

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$$\cosh k_i(\eta+h)\ \mathrm{e}^{\mathrm{i}k_i|x-\xi|},$$

$$\mathcal{G}(x,y;\xi,\eta) = \log(\frac{r}{h}) + \log(\frac{r_2}{h}) +$$

$$+\int_{\mathcal{L}'} \left\{ \frac{\gamma+K}{\gamma} \frac{\mathbf{e}^{-\gamma h}\cosh\gamma(y+h)\cosh\gamma(\eta+h)\cos\gamma(x-\xi)}{K\cosh\gamma h - \gamma\sinh\gamma h} + \frac{\mathbf{e}^{-\gamma h}}{\gamma} \right\} \mathrm{d}\gamma,$$

where r_2 is the distance to mirror of the source.

The equation for the deflection $w(x)$ of the platform becomes

$$2\pi \left(\mathcal{D}\frac{\partial^4}{\partial x^4} - \mu + 1 \right) w(x) + K \int_0^l \mathcal{G}(x,0;\xi,0) \left\{ \mu - \mathcal{D}\frac{\partial^4}{\partial \xi^4} \right\} w(\xi)\,\mathrm{d}\xi =$$

$$= -2\pi \mathrm{i} \sum_{i=0}^{\infty} \frac{1}{k_i} \frac{k_i^2 - K^2}{hk_i^2 - hK^2 + K} \cosh k_i h \cosh k_i(y_0 + h)\ \mathrm{e}^{\mathrm{i} k_i (x - x_0)}.$$

Here we have used the mode expansion of the source at (x_0, y_0).

The equation for the deflection $w(x)$ of the platform becomes

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$$=-2\pi\mathsf{i}\sum_{i=0}^{\infty}\frac{1}{k_i}\frac{k_i^2-K^2}{hk_i^2-hK^2+K}\cosh k_ih\cosh k_i(y_0+h)\ \mathsf{e}^{\mathsf{i}k_i(x-x_0)}.$$

Here we have used the mode expansion of the source at (x_0,y_0).

We introduce the following expansion for $w(x)$

$$w(x)=\sum_{n=0}^{N+1}\left(a_n\mathbf{e}^{\mathsf{i}\kappa_n x}+b_n\mathbf{e}^{-\mathsf{i}\kappa_n(x-l)}\right)$$

We use the first version of the Green's function. Integration with respect to ξ and closure of the contour in the complex γ-plane leads to the dispersion relation:

$$(\mathcal{D}\kappa^4 - \mu + 1)\kappa \tanh \kappa h = K$$

together with a $2N$ equations of the $2N+4$ unknowns a_n and b_n:

$$\sum_{n=0}^{N+1}(\mathcal{D}\kappa^4-\mu)\left[\frac{a_n}{\kappa_n-k_i}-\frac{b_n\mathbf{e}^{\mathsf{i}\kappa_n l}}{\kappa_n+k_i}\right]=-\delta_n^0$$

$$\sum_{n=0}^{N+1}(\mathcal{D}\kappa^4-\mu)\left[\frac{-a_n\mathbf{e}^{\mathsf{i}\kappa_n l}}{\kappa_n+k_i}+\frac{b_n}{\kappa_n-k_i}\right]=0.$$

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We also have two conditions at each end of the platform:

$$\frac{\mathrm{d}^2 w}{\mathrm{d}x^2}=\frac{\mathrm{d}^3 w}{\mathrm{d}x^3}=0 \quad \text{at } x=0,l$$

For a source (strength$=\frac{g}{\mathrm{i}\omega}$) on the left hand side of the platform $x_0 < 0$

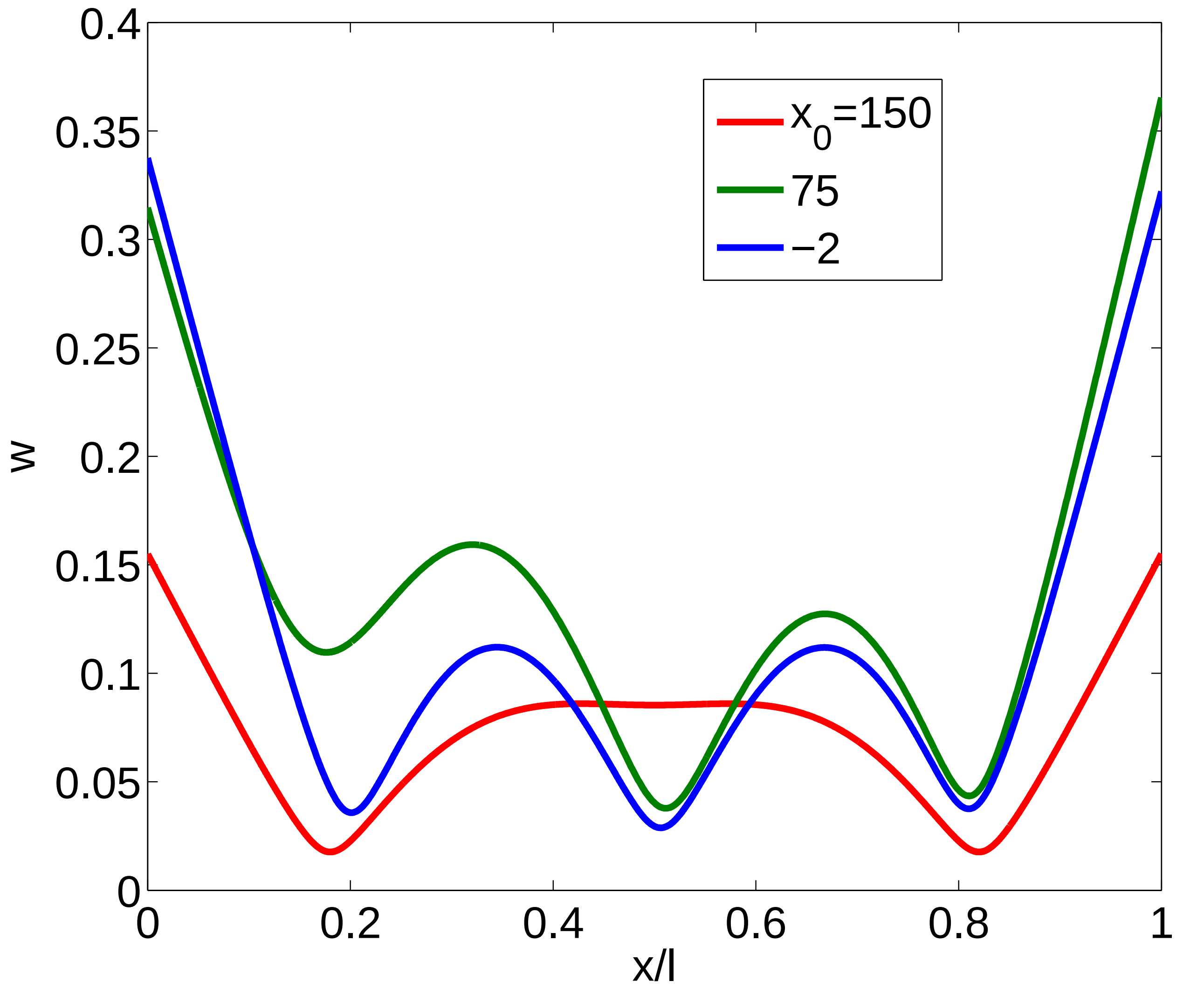
$$\sum_{n=0}^{N+1} (\mathcal{D}\kappa_n^4 - \mu) \left[\frac{a_n}{\kappa_n - k_i} - \frac{b_n \mathbf{e}^{\mathrm{i}\kappa_n l}}{\kappa_n + k_i} \right] =$$

$$= \frac{k_i^2 - K^2}{k_i^2 K} \cosh k_i h \cosh k_i (y_0 + h) \mathbf{e}^{-\mathrm{i} k_i x_0}$$

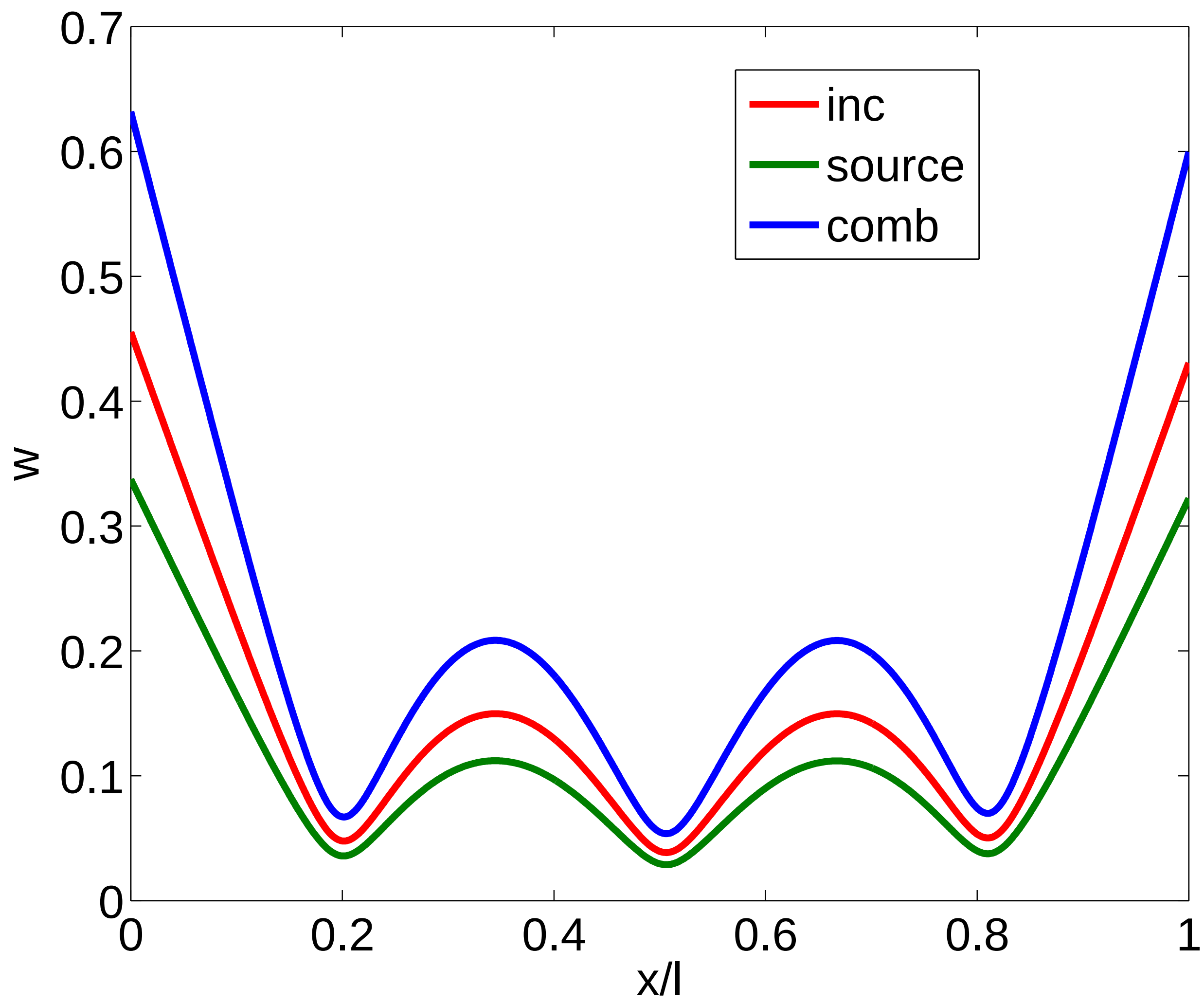
$$\sum_{n=0}^{N+1} (\mathcal{D}\kappa_n^4 - \mu) \left[\frac{-a_n \mathbf{e}^{\mathrm{i}\kappa_n l}}{\kappa_n + k_i} + \frac{b_n}{\kappa_n - k_i} \right] = 0.$$

If the source is underneath the platform we split the platform in two parts and obtain a set of equations in a similar way.

Variable x_0, fixed $y_0 = -1.5, l = 300$ m. , $\mathcal{D} = 10^7$ m.^{4} and $\frac{\omega^2}{g} = 2\pi/60$



Combination of source and incident field, $x_0 = -2,\ y_0 = -1.5$



$$\left(\begin{array}{c:c} \mathcal{C} \to \mathcal{C} & \mathcal{P} \to \mathcal{C} \\ np \text{ x } np & (2N+4) \text{ x } np \\ \hdashline \mathcal{C} \to \mathcal{P} & \mathcal{P} \to \mathcal{P} \\ np \text{ x } (2N+4) & (2N+4) \text{ x } (2N+4) \end{array}\right)$$

The circle $\mathcal{C}$ we have divided in np segments, so we have np unknown values of the source strengths. We use the third expression for the Green's function.

At the platform $\mathcal{P}$ we have the matrix as before for the $2N+4$ unknown coefficient of the series.

For $n = 0, \cdots, N+1$ the effect of $\mathcal{P}$ on $\mathcal{C}$ becomes

$$-\frac{2\pi K}{\mathcal{D}} \sum_{i=0}^{N-1} (\mathcal{D}\kappa_n^4 - \mu) k_i \frac{\cosh k_i(y+h)}{(k_i^2 h + K - K^2 h)\cosh k_i h} \, \mathrm{e}^{-\mathrm{i}k_i x} .$$

$$\cdot \left[a_n \frac{\mathrm{e}^{\mathrm{i}(\kappa_n + k_i)l} - 1}{\kappa_n + k_i} + b_n \frac{\mathrm{e}^{\mathrm{i}\kappa_n l} - \mathrm{e}^{\mathrm{i}k_i l}}{\kappa_n - k_i} \right]$$

The influence of unknown source strength on $\mathcal{C}$ on the platform $\mathcal{P}$ for $i = 0, \cdots, N-1$ follows from

$$\int_{\mathcal{C}} \phi(\xi,\eta) \frac{\partial \mathcal{G}_i(x,z;\xi,\eta)}{\partial n}\, \mathrm{d}s$$

with

$$\mathcal{G}_i(x,y;\xi,\eta) = -\frac{1}{k_i} \frac{k_i^2 - K^2}{hk_i^2 - hK^2 + K} \cdot$$

$$\cdot \cosh k_i(y+h) \cosh k_i(\eta+h)\ \mathrm{e}^{\mathrm{i}k_i|x-\xi|},$$

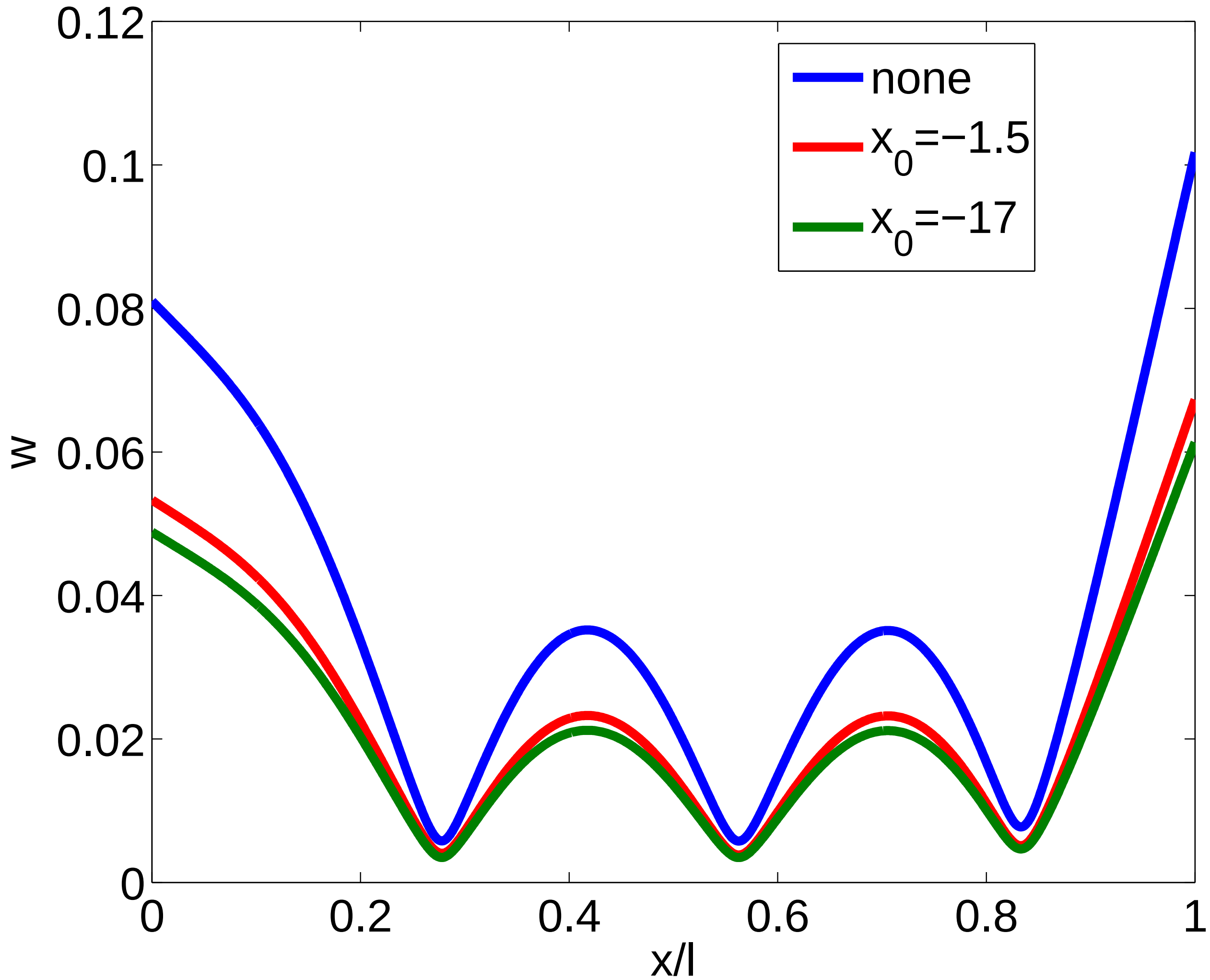
The motion of the cylinder influences the righthand side of $\mathcal{P}$ by means of the term

$$\int_{\mathcal{C}} \frac{\partial \phi(\xi,\eta)}{\partial n} \mathcal{G}_i(x,z;\xi,\eta)\, \mathsf{d}s$$

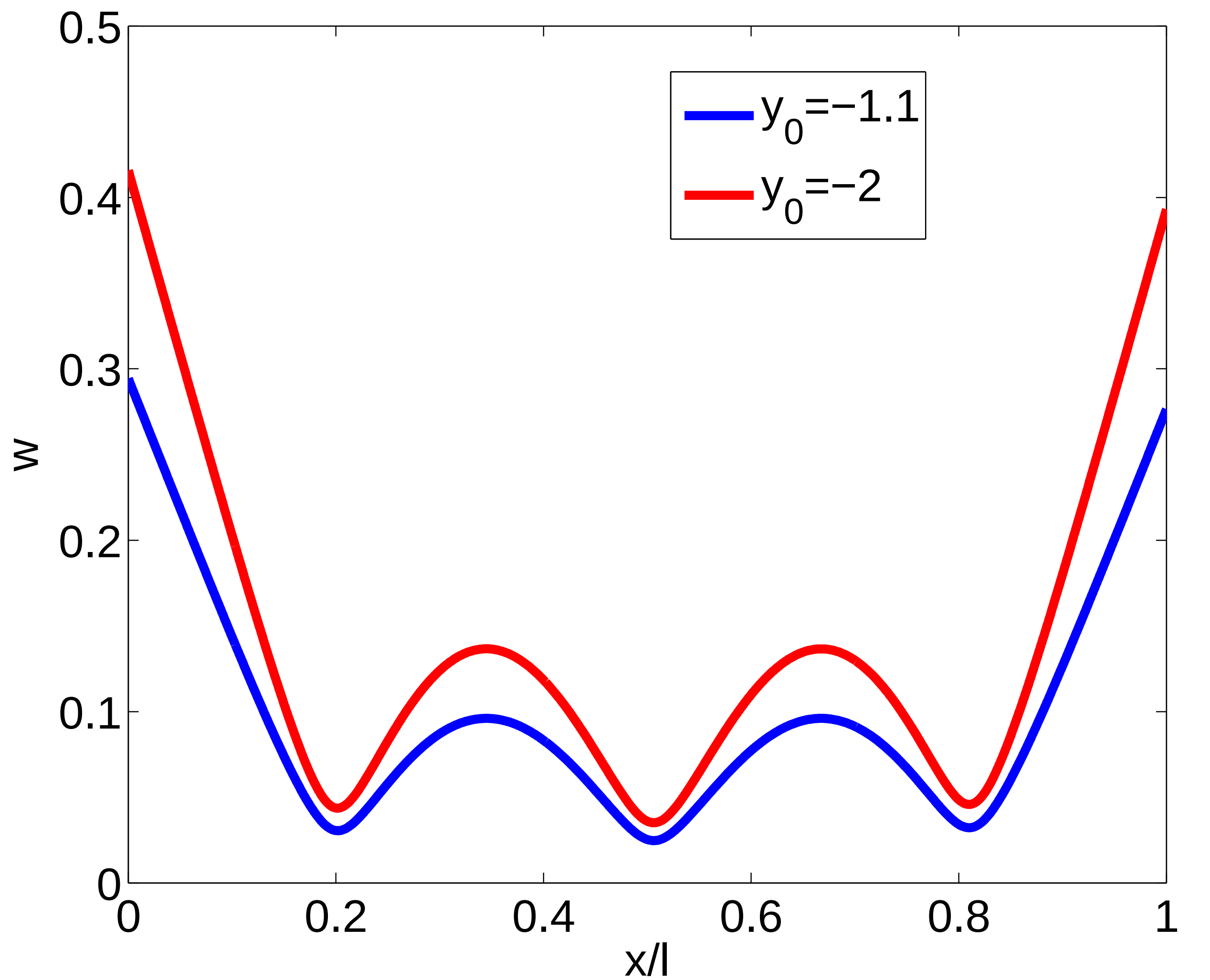
for $i = 1, \cdots N-1$ and

$$\mathcal{G}_i(x,y;\xi,\eta) = -\frac{1}{k_i}\frac{k_i^2 - K^2}{hk_i^2 - hK^2 + K}\cdot$$

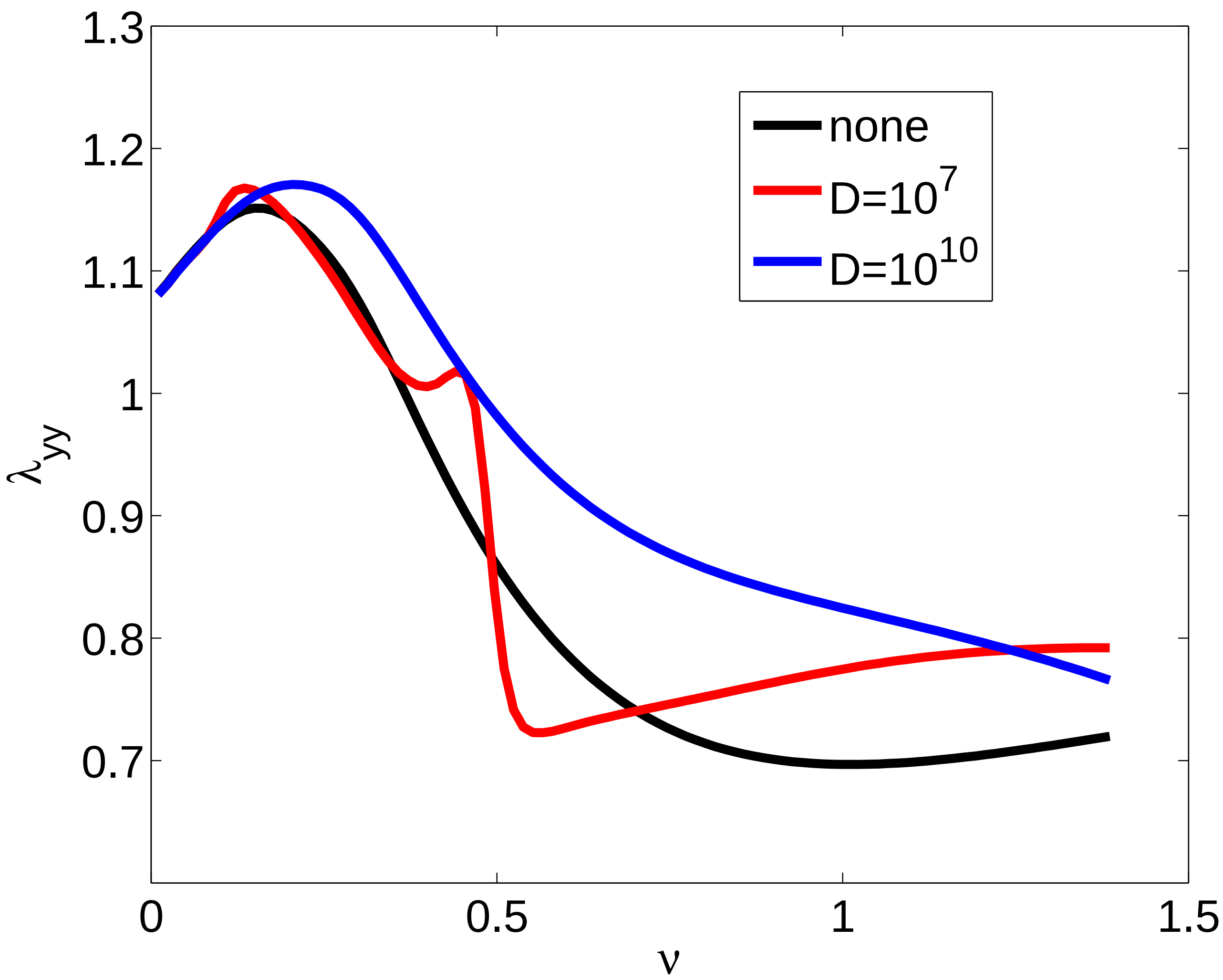
$$\cdot \cosh k_i(y+h) \cosh k_i(\eta+h)\ \mathsf{e}^{\mathsf{i}k_i|x-\xi|},$$


0.12
0.1
0.08
0.06
0.04
0.02
0
w
0
0.2
0.4
0.6
0.8
1
x/l
none
x_0=−1.5
x_0=−17

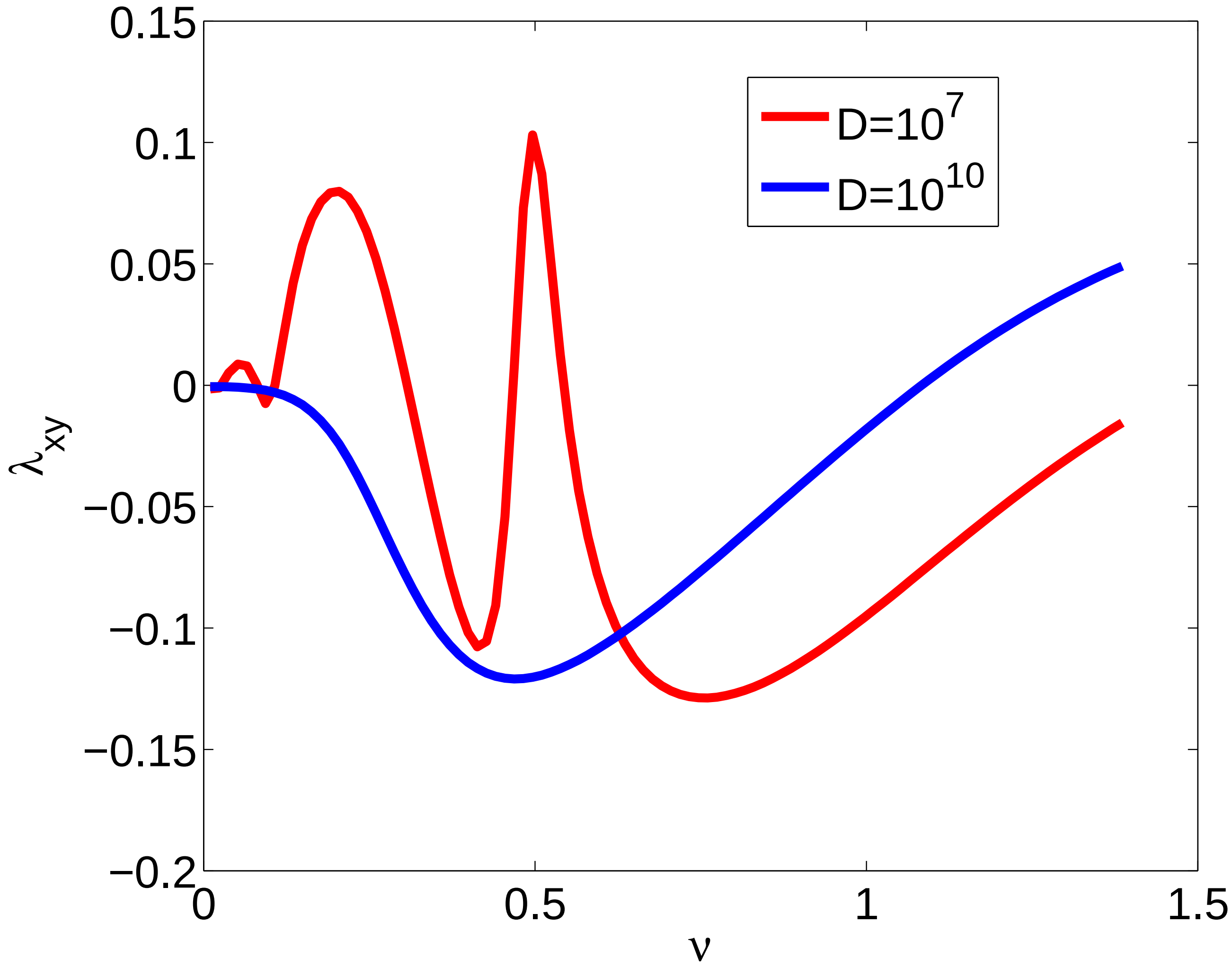
deflection with fixed $x_0 = -1.5$ and $\frac{\omega^2}{g} = 2\pi/60$



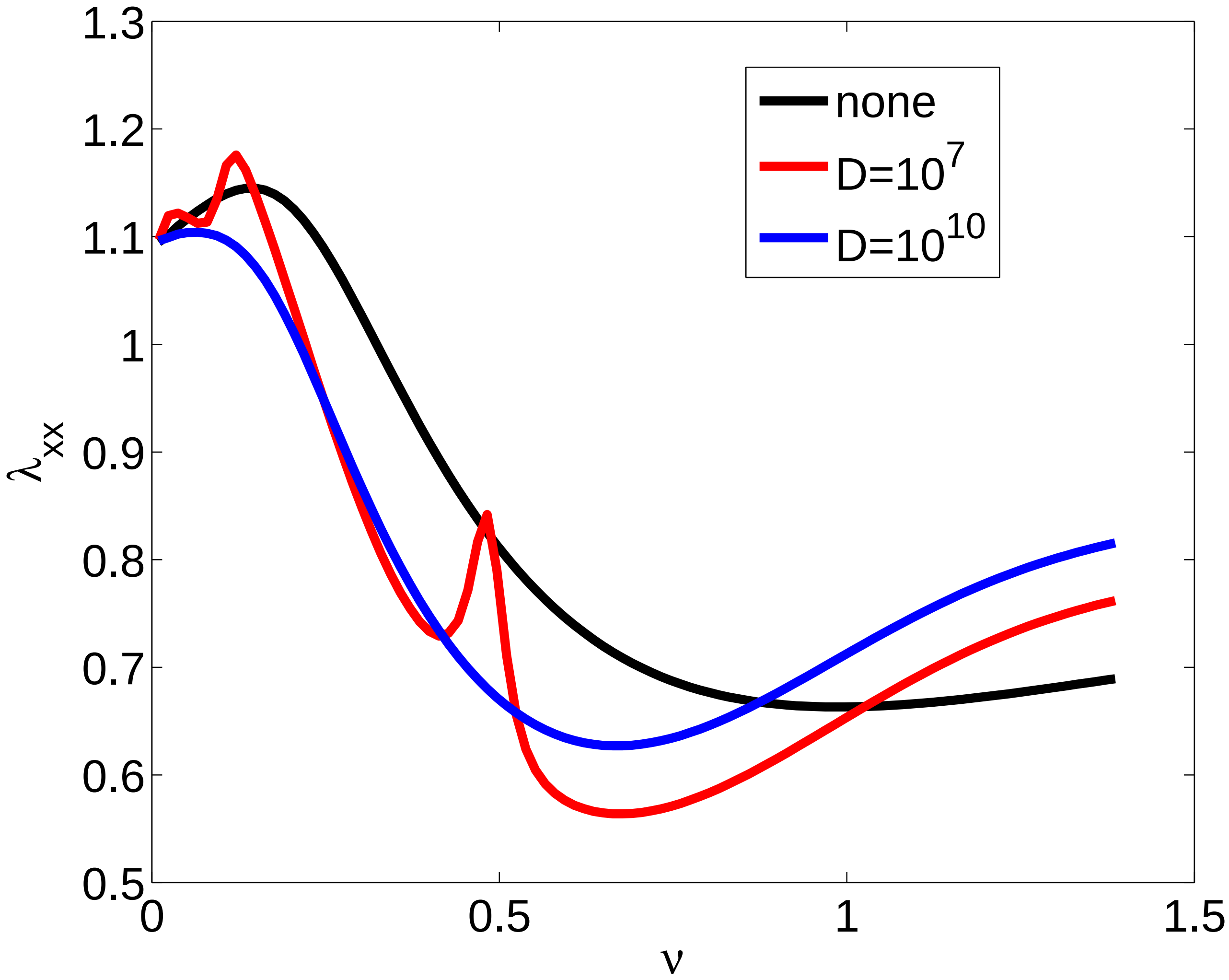
Added mass in heave (λ_{yy}),with and without $\mathcal{P}$, $x_0 = -1.5,\ y_0 = -2$



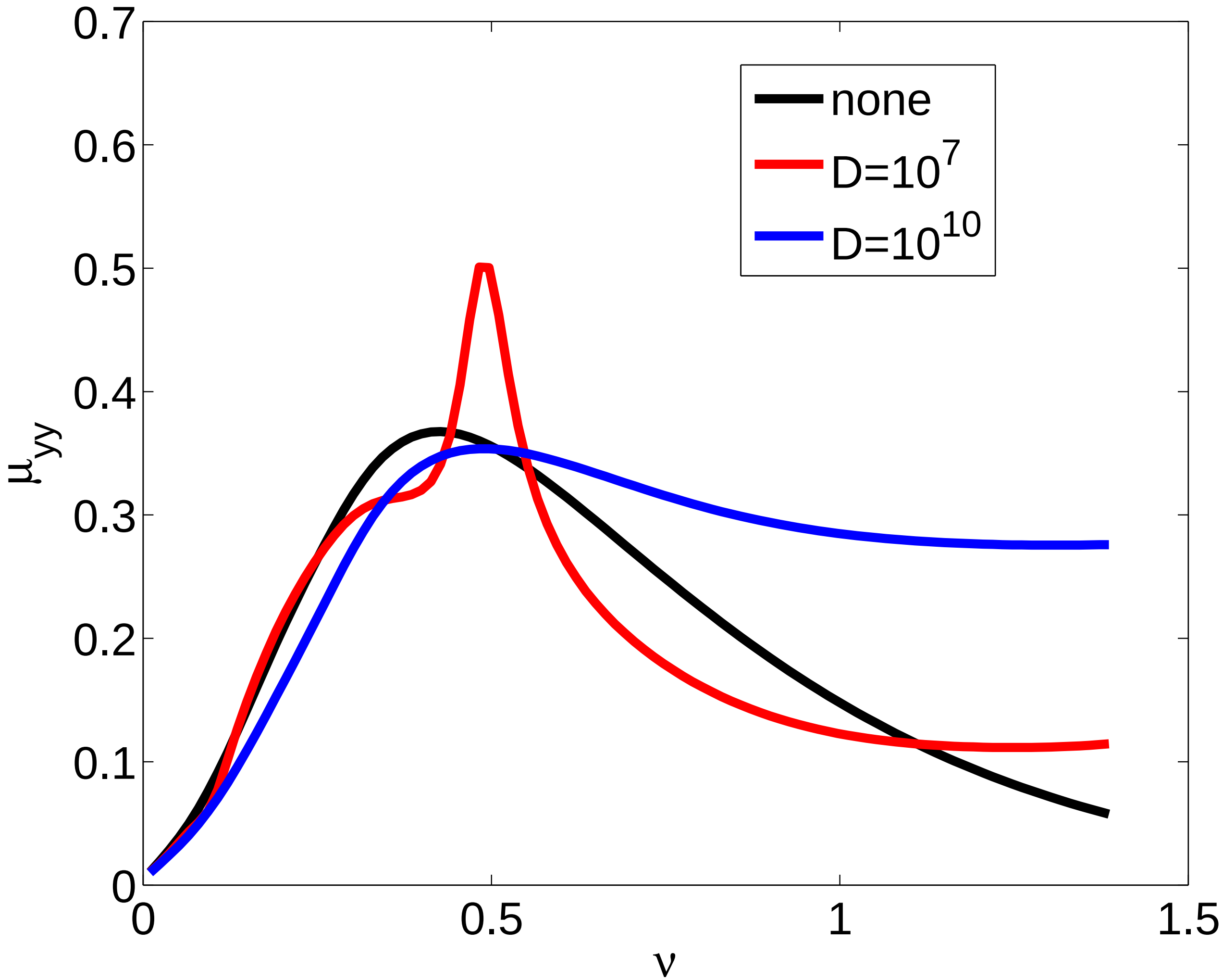
Added mass (coupling coefficient) (λ_{xy}), $x_0 = -1.5$, $y_0 = -2$



Added mass in surge (λ_{xx}), with and without $\mathcal{P}$, $x_0 = -1.5,\ y_0 = -2$



Damping in heave (μ_{yy}),with and without $\mathcal{P}$, $x_0 = -1.5,\ y_0 = -2$



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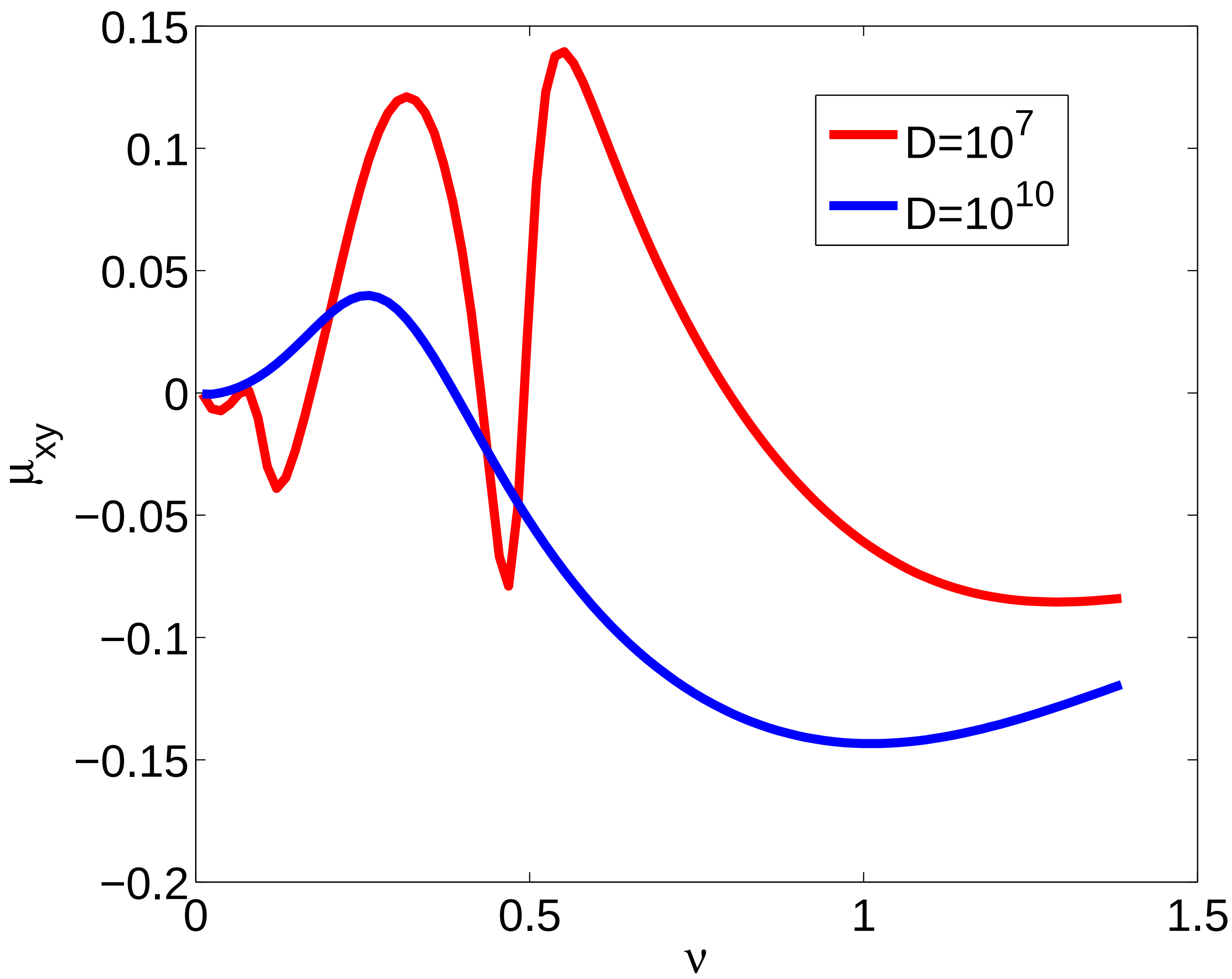
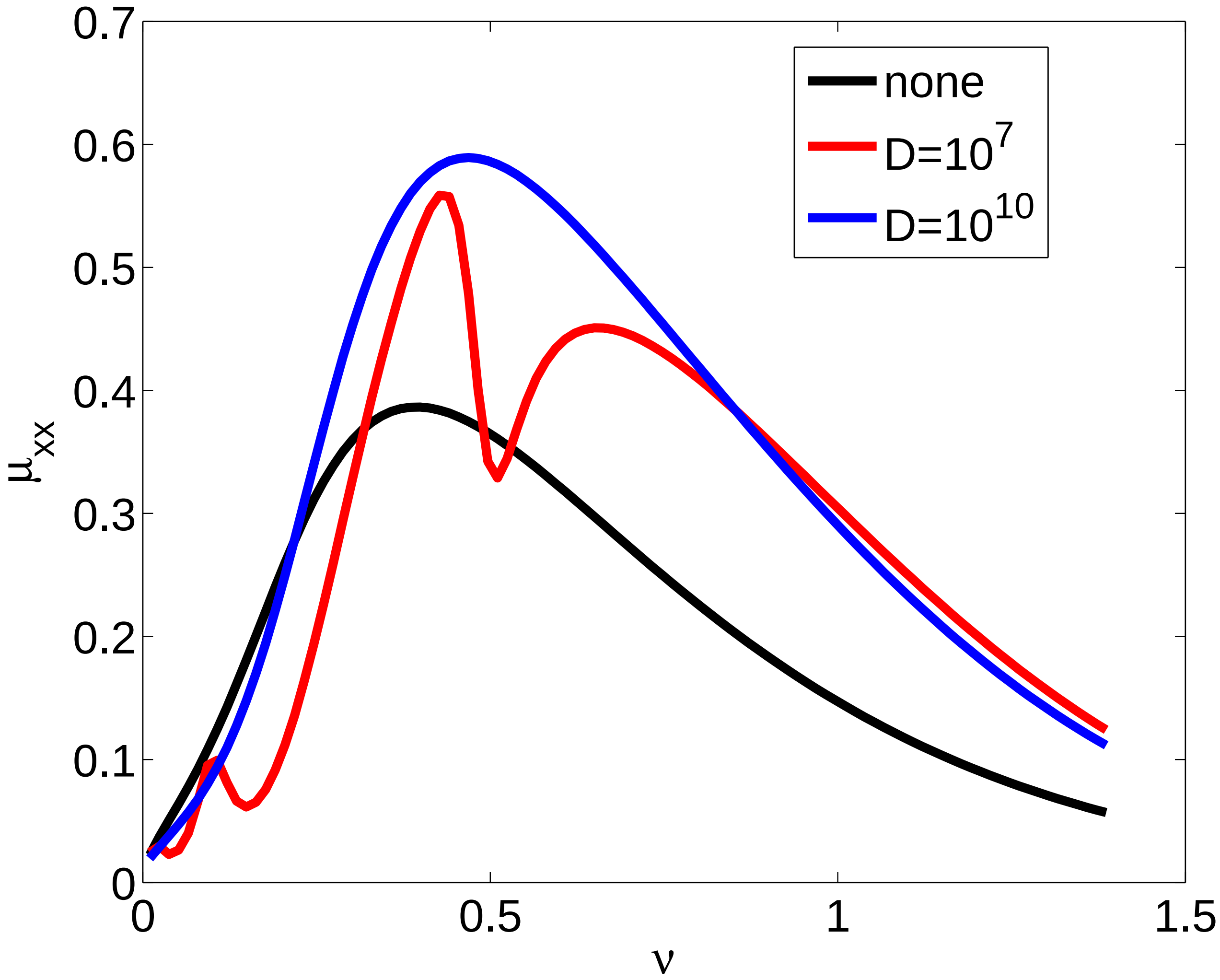


Damping in surge (μ_{xx}),with and without $\mathcal{P}$, $x_0 = -1.5,\ y_0 = -2$



Two suggestions for extension of the method for $\mathcal{C}$ underneath $\mathcal{P}$:

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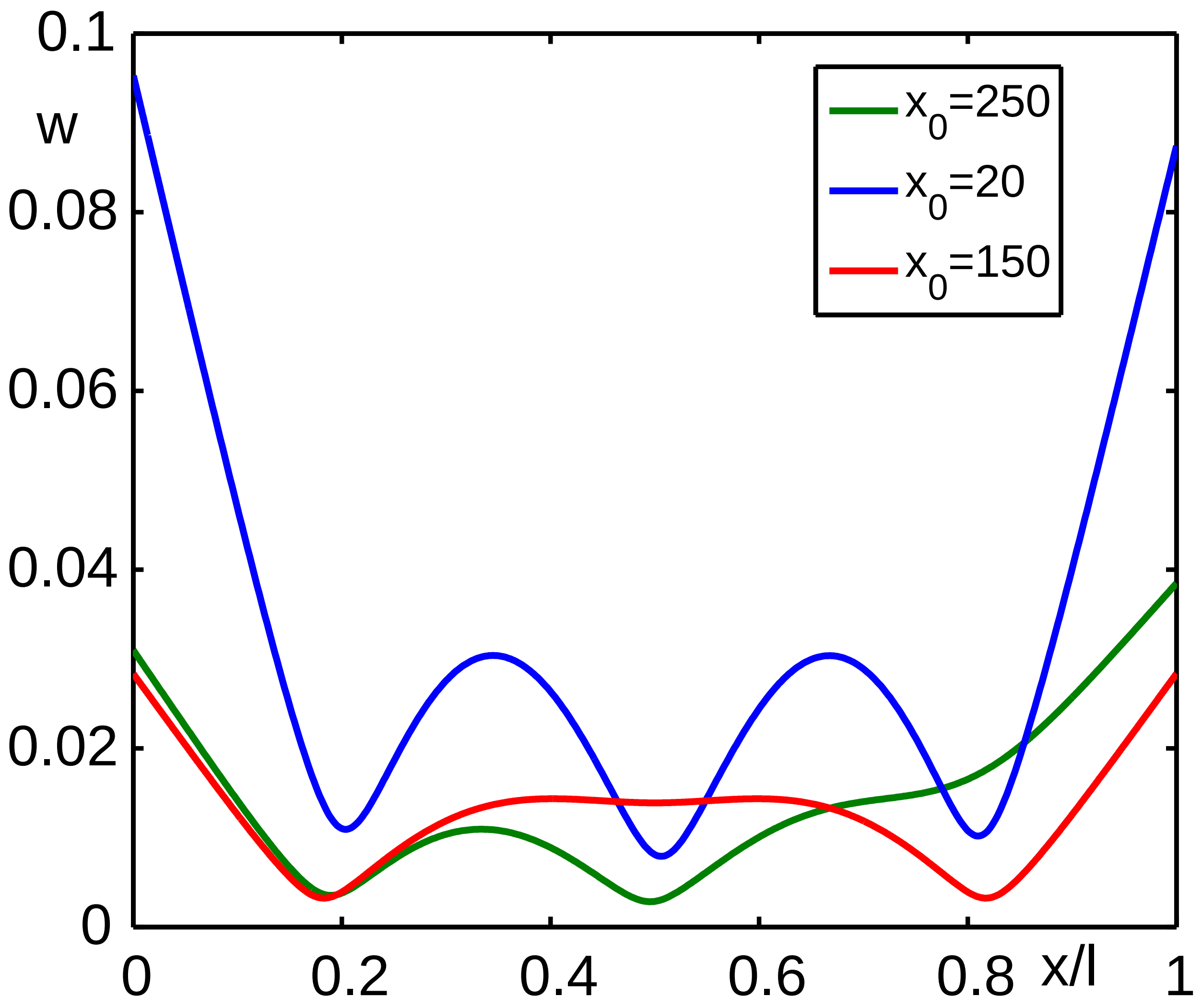
- One may make use of the fact that the dimension (in the $x-$direction) of the cylindrical object is small compared to the length of the platform. The same splitting procedure as described in the case of the point source may be used.

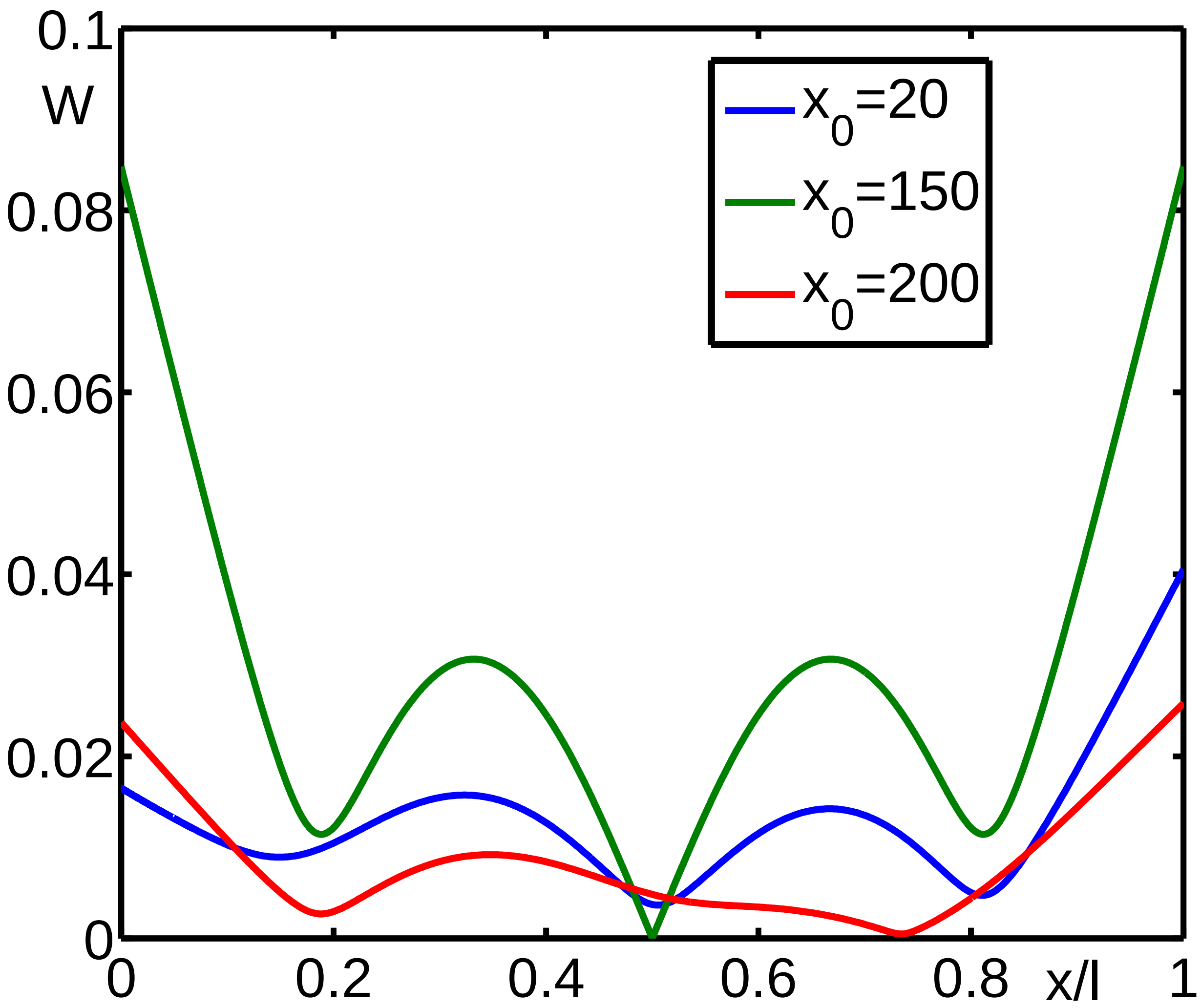
Two suggestions for extension of the method for $\mathcal{C}$ underneath $\mathcal{P}$:

- One may make use of the fact that the dimension (in the $x-$direction) of the cylindrical object is small compared to the length of the platform. The same splitting procedure as described in the case of the point source may be used.
- For a large object or small platform one may solve the plate problem in a way as described in reference (1). I suggest to use the expansion in orthogonal modes, because then the integration with respect to x along the platform may be carried out analytically. The resulting integrals can be computed numerically as described in the reference.

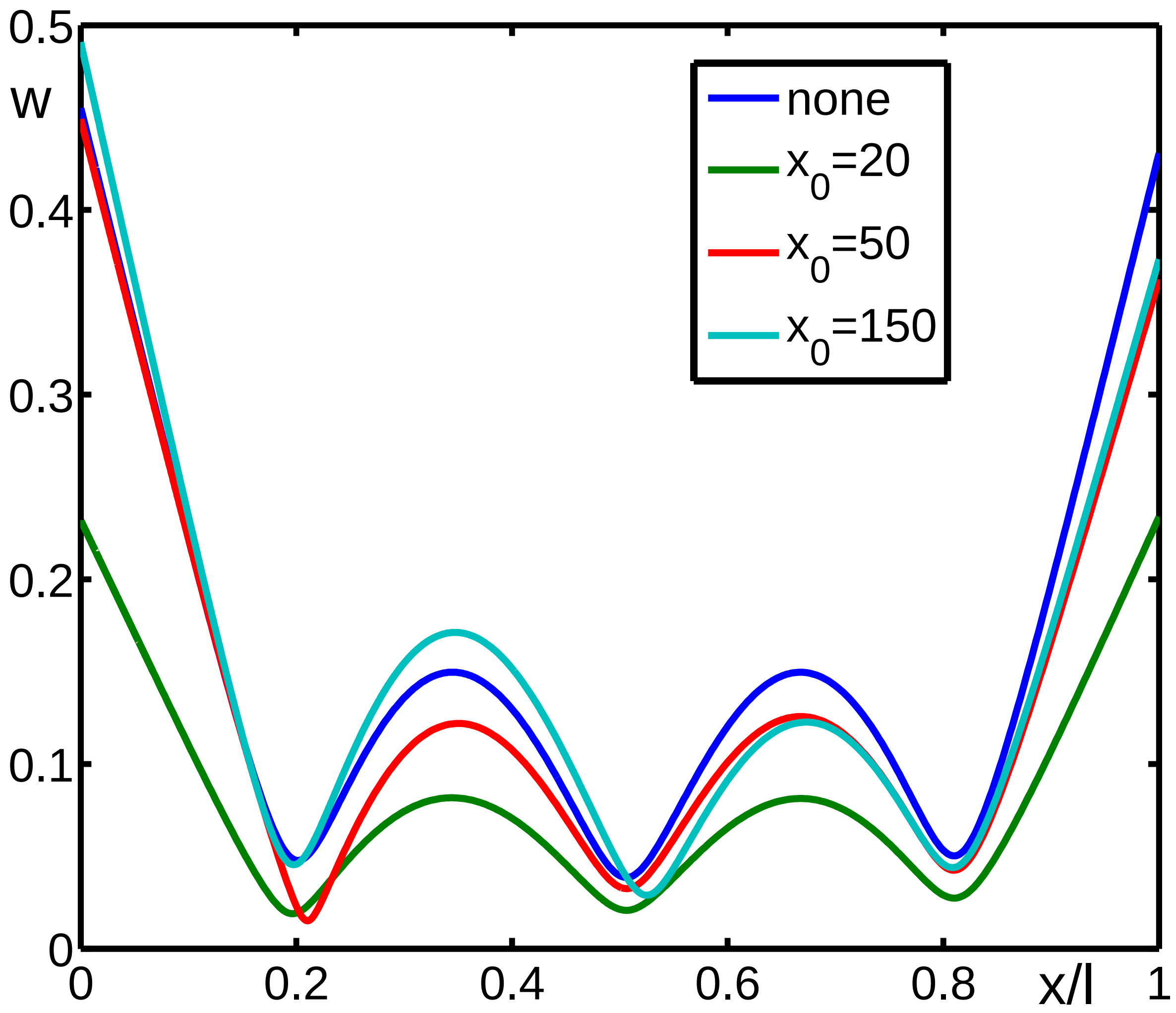
Effect of a heaving cylinder on the deflection $y_0 = -2,\ \frac{\omega^2}{g} = 2\pi/60)$



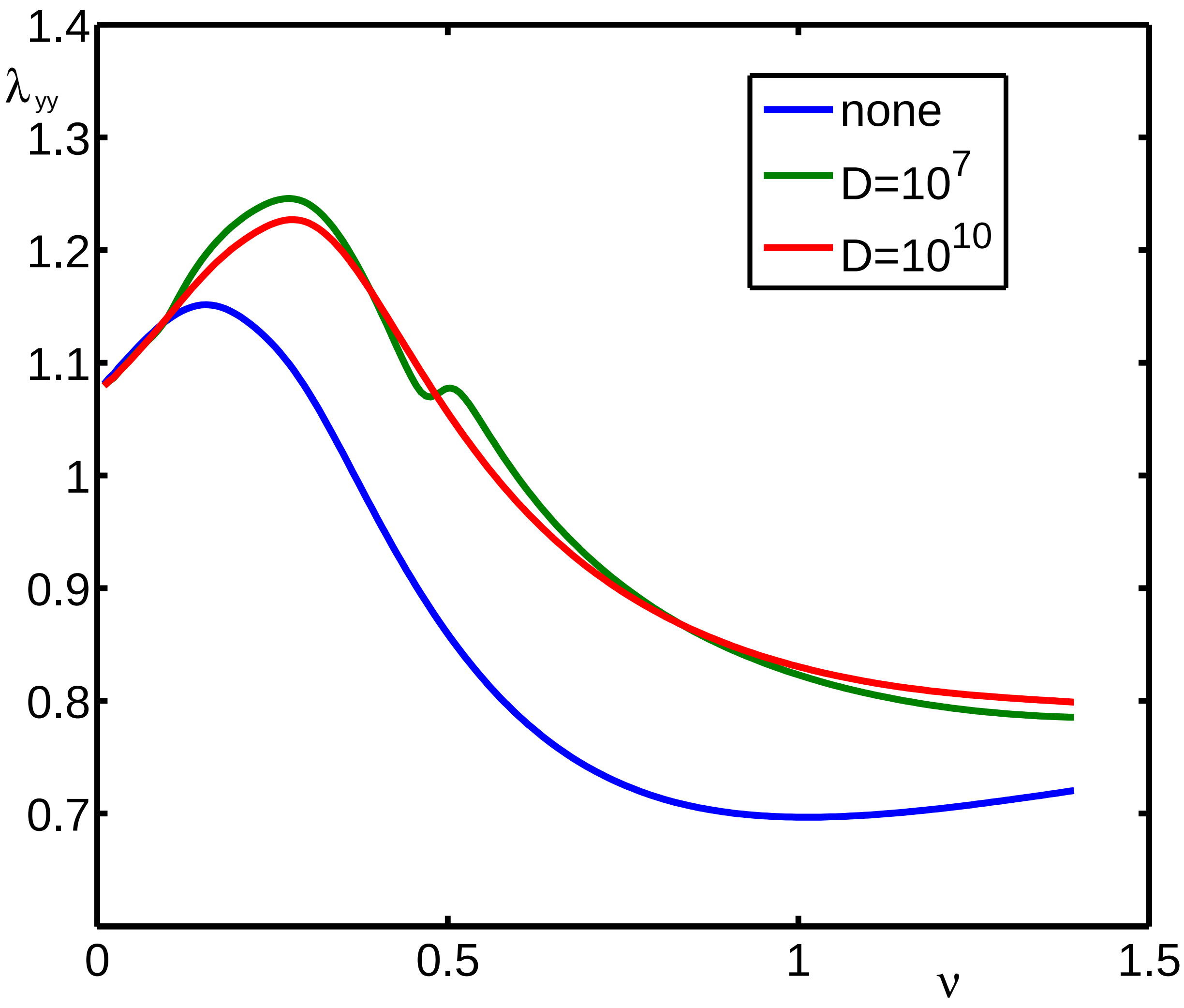
Effect of a surging cylinder on the deflection $y_0 = -2,\ \frac{\omega^2}{g} = 2\pi/60$



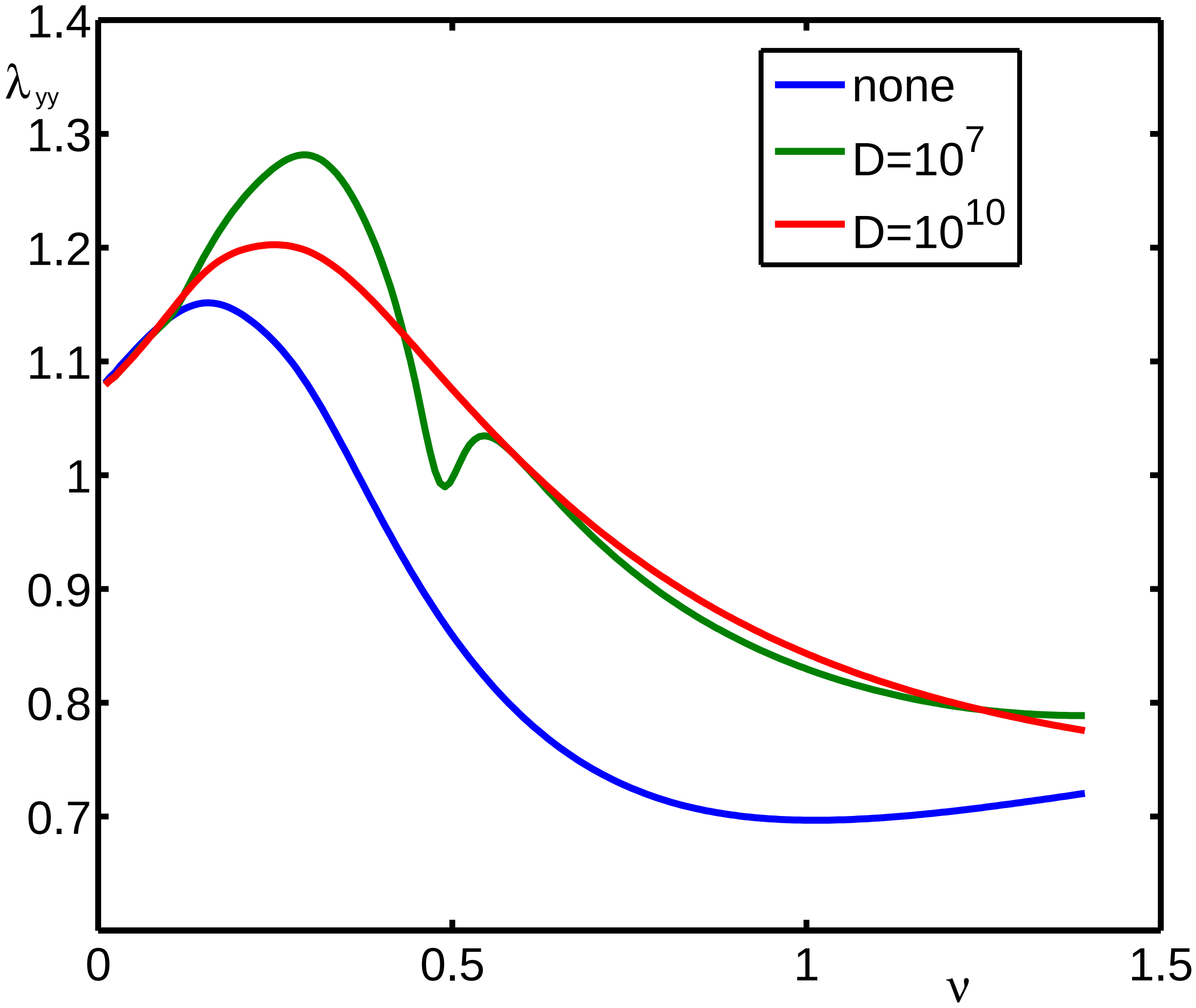
Deflection for an incident wave $y_0 = -1.1,\ \frac{\omega^2}{g} = 2\pi/60)$



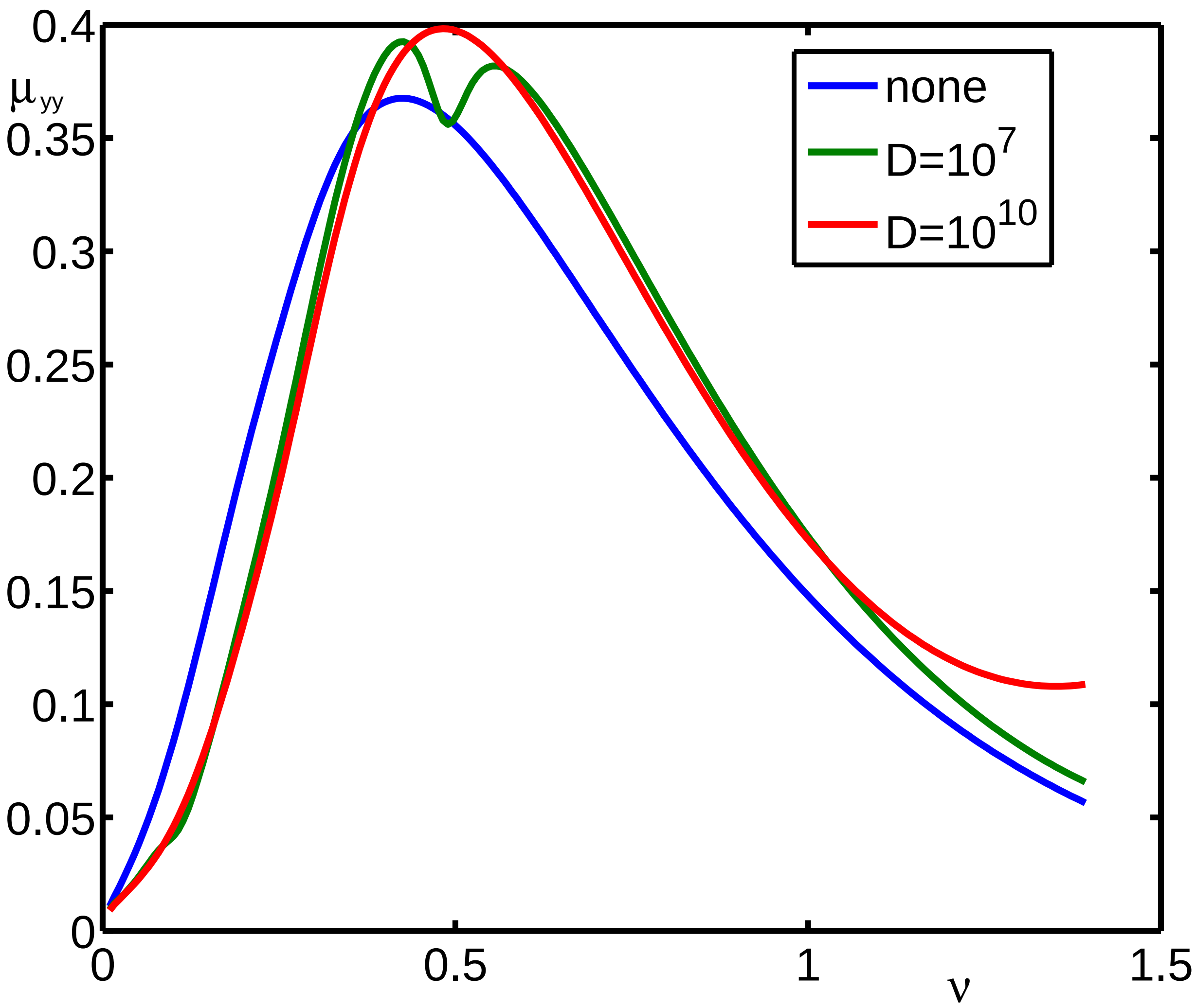
Added mass in heave, $x_0 = 2,\ y_0 = -2$



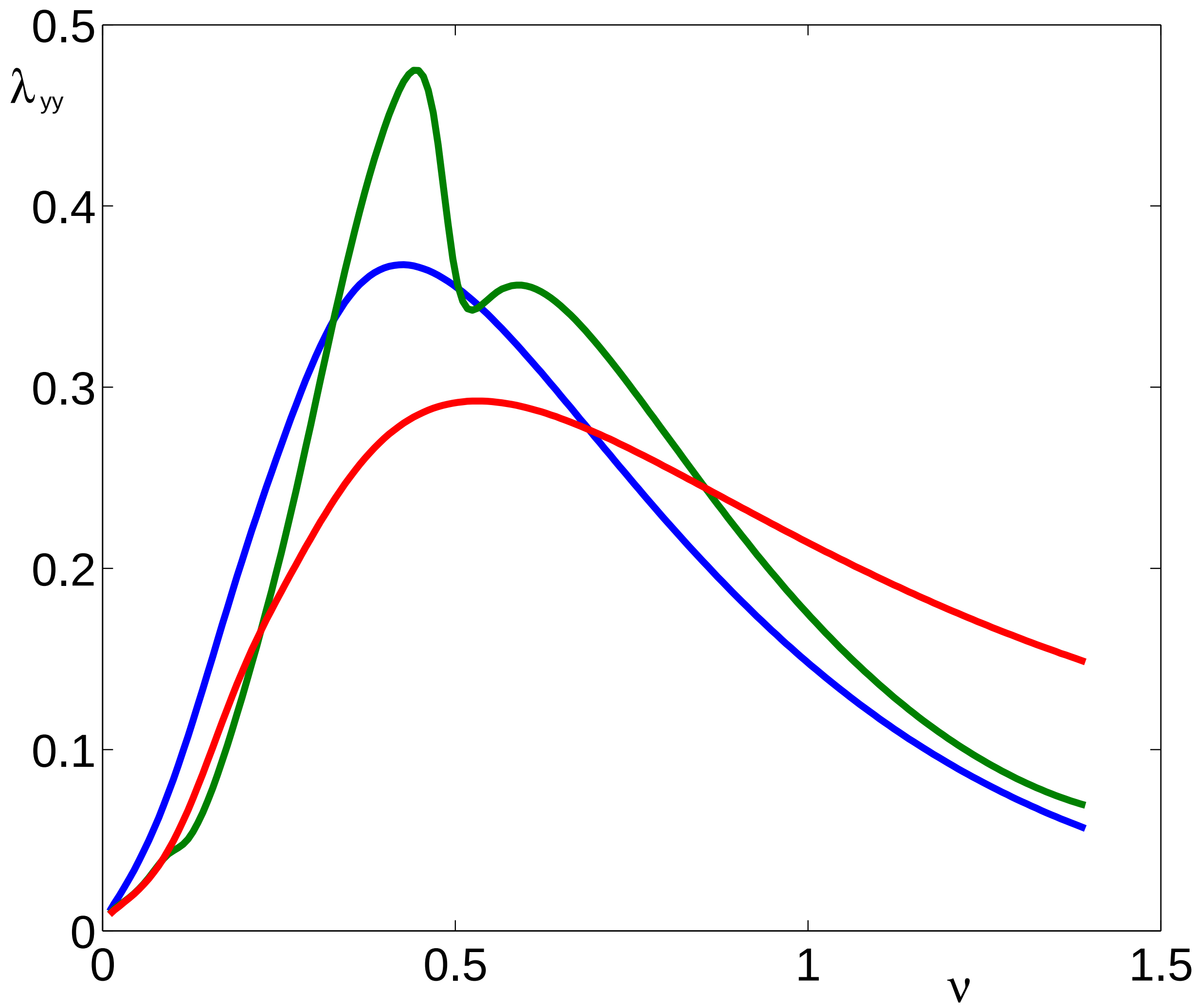
Added mass in heave, $x_0 = 10, \ y_0 = -2$

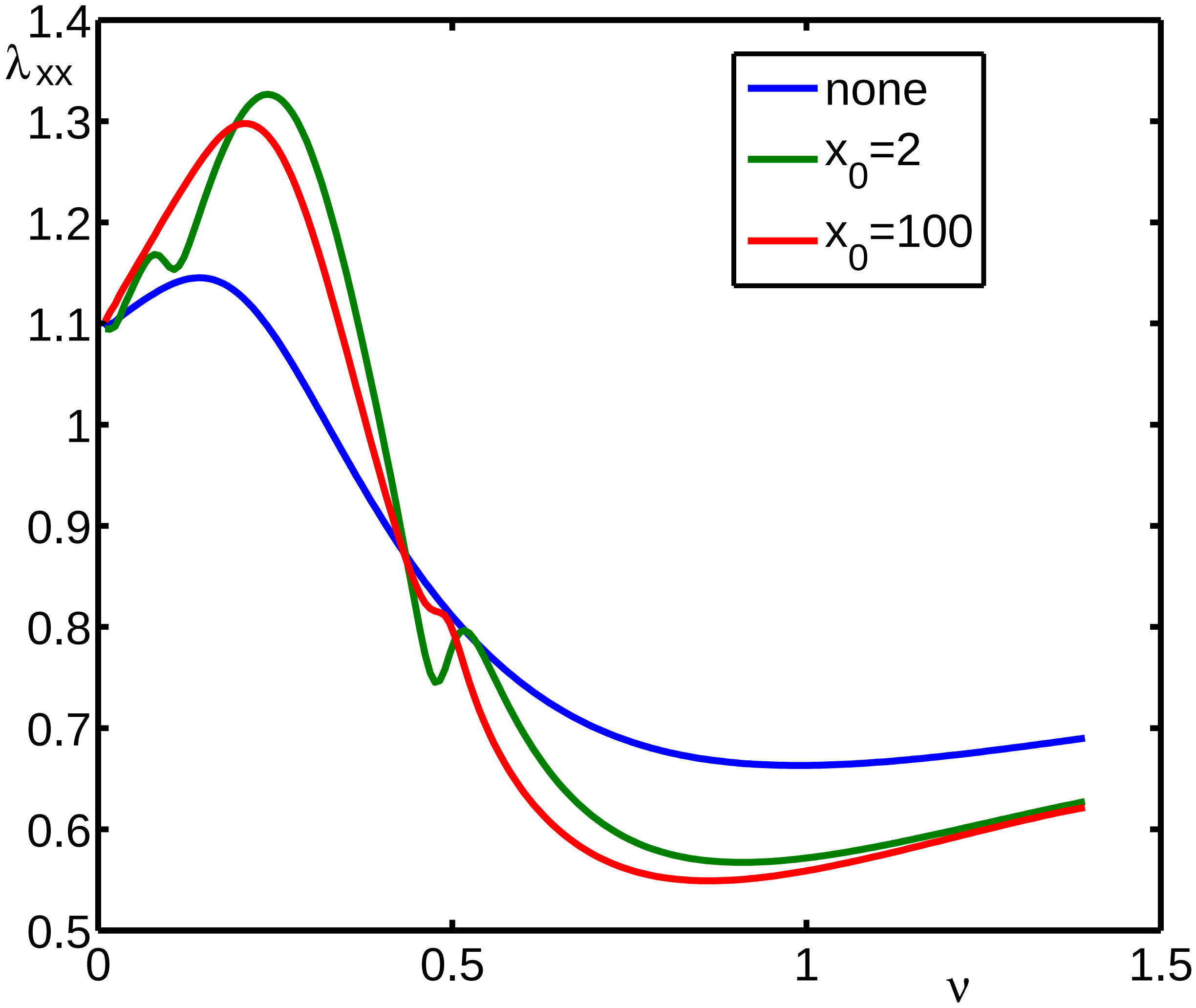


Damping in heave, $x_0 = 2,\ y_0 = -2$

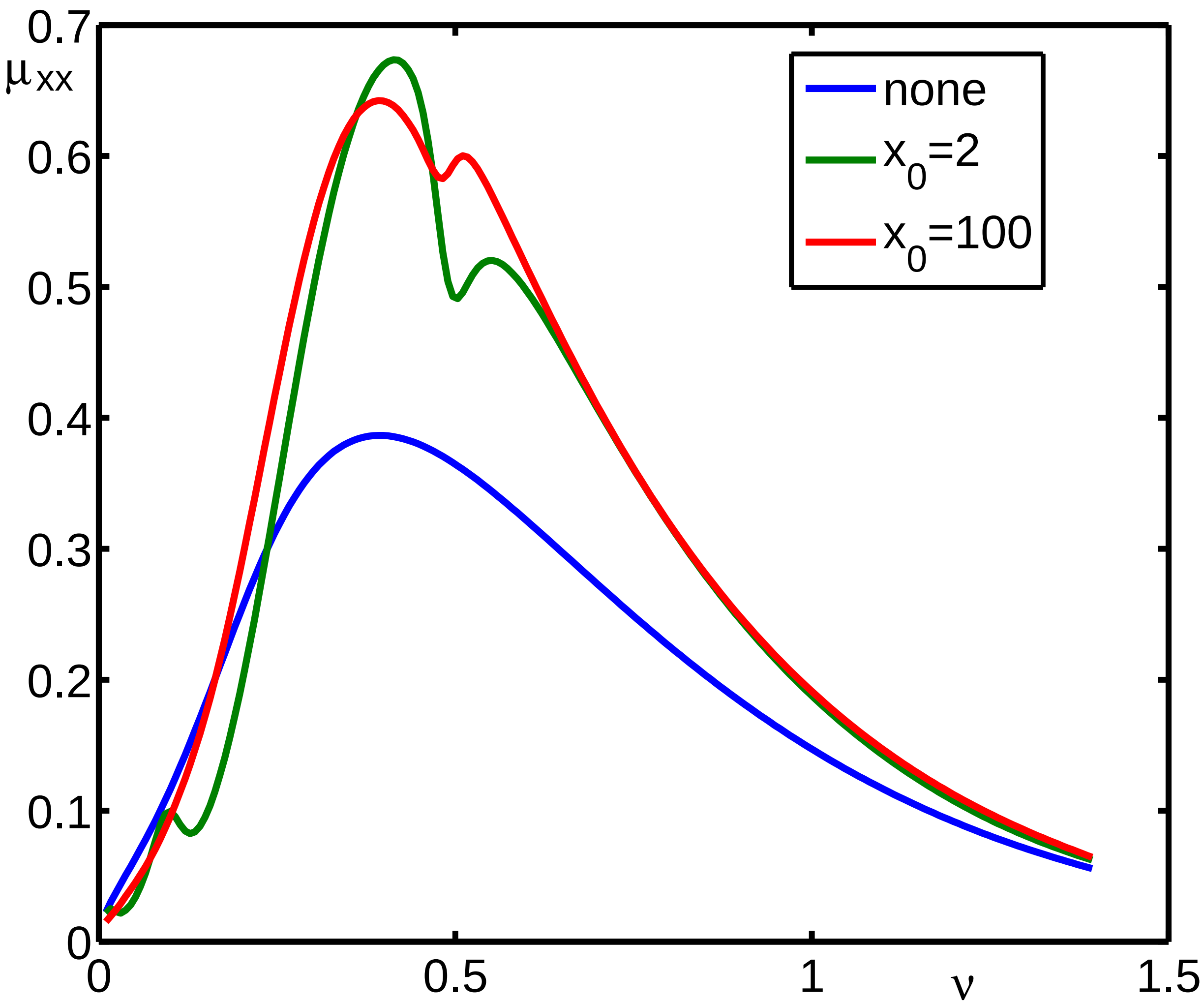


Damping in heave, $x_0 = 10,\ y_0 = -2$

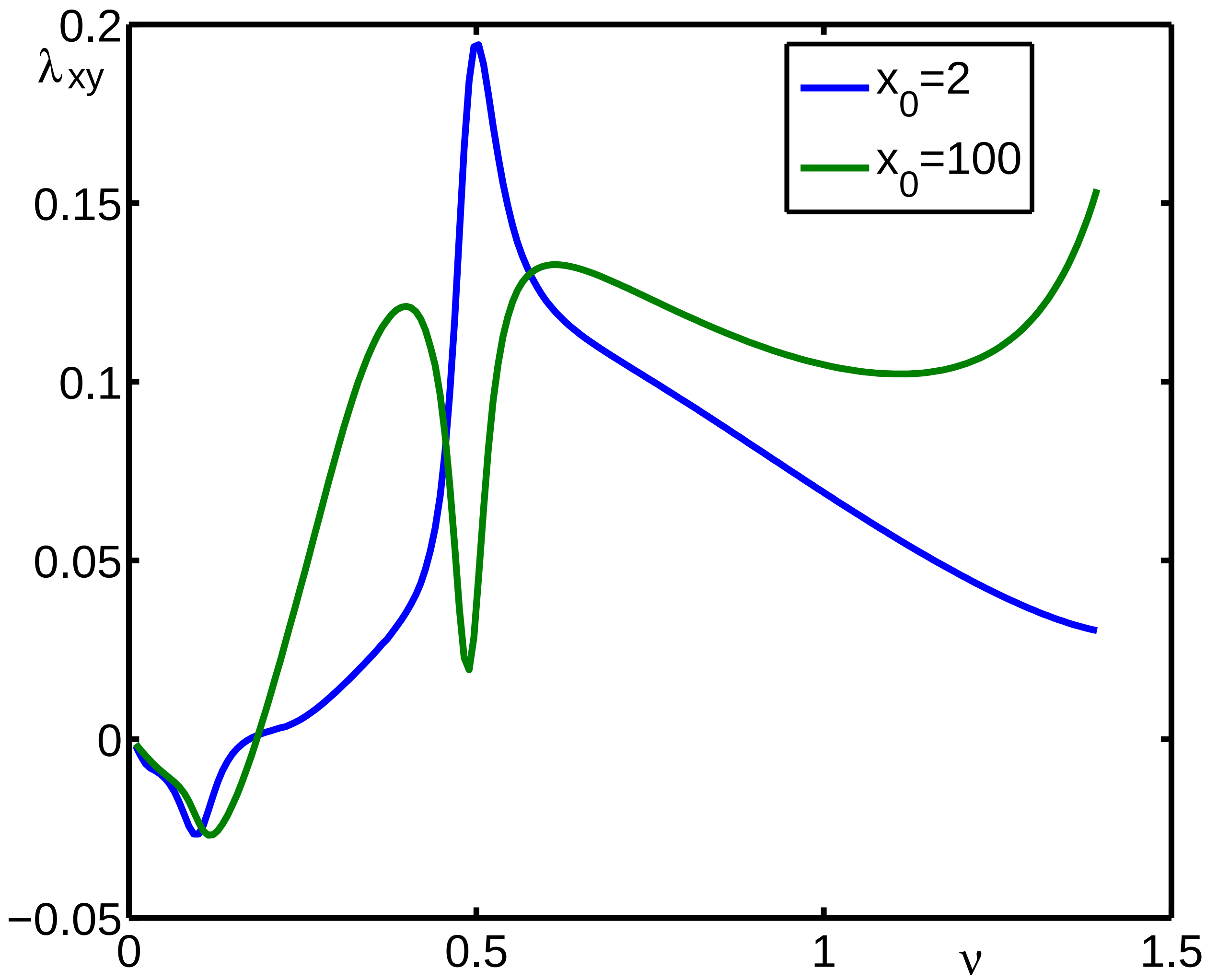



1.4
1.3
1.2
1.1
1
0.9
0.8
0.7
0.6
0.5
λxx
0
0.5
1
1.5
ν
none
x0=2
x0=100

Damping in surge, $y_0 = -2,\ \mathcal{D} = 10^7$



Added mass in surge-heave, $y_0 = -2$, $\mathcal{D} = 10^7$



Damping in surge-heave, $y_0 = -2$, $\mathcal{D} = 10^7$