Added Resistance by means of time-domain models in seakeeping

Aad J. Hermans,
Email: A.J.Hermans@ewi.TUDelft.nl,
Department Applied Mathematical Analysis ,
Faculty of Electrical Engineering, Mathematics and Computer Science,
Delft University of Technology,
The Netherlands.

Non-linear formulation

Potential theory $\Delta\Phi = 0$

Free-surface condition (no surface tension) on $z = \zeta(x, y)$

$$g\frac{\partial\Phi}{\partial z} + \frac{\partial^2\Phi}{\partial t^2} + \nabla\Phi\cdot\nabla\frac{\partial\Phi}{\partial t} +$$

$$\left(\frac{\partial\Phi}{\partial x}\frac{\partial}{\partial x_\zeta} + \frac{\partial\Phi}{\partial y}\frac{\partial}{\partial y_\zeta}\right)\left(\frac{\partial\Phi}{\partial t} + \frac{1}{2}\nabla\Phi\cdot\nabla\Phi\right) = 0$$

where

$$\frac{\partial F(x, y, \zeta(x, y))}{\partial x_\zeta} = \frac{\partial F}{\partial x} + \frac{\partial F}{\partial z}\frac{\partial\zeta}{\partial x}$$

is differentiation along the free-surface.

Non-linear formulation

At the exact hull $H(t)$ we have

$$\frac{\partial \Phi}{\partial n} = \frac{\partial \boldsymbol{\alpha}}{\partial t} \cdot \mathbf{n},$$

where $\boldsymbol{\alpha}$ is the displacement in the mean ship-fixed coordinate system.

Radiation condition.

TUDelft

Decomposition of the potential

We decompose the potential function in a steady and unsteady part

$$\Phi(\mathbf{x},t) = \Phi_s(\mathbf{x}) + \phi_u(\mathbf{x},t)\,.$$

Choices for the steady part are

- $\Phi_s(\mathbf{x}) = Ux$ the unperturbed flow
- $\Phi_s(\mathbf{x}) = \Phi_r(\mathbf{x})$ double body flow
- $\Phi_s(\mathbf{x}) = \Phi_r(\mathbf{x}) + \phi(\mathbf{x})$
- Solution of the nonlinear steady problem (RAPID)

The steady potential

The non-linear free surface conditions for the steady potential are

$$\frac{\partial\Phi_s}{\partial x}\frac{\partial\zeta_s}{\partial x}+\frac{\partial\Phi_s}{\partial y}\frac{\partial\zeta_s}{\partial y}-\frac{\partial\Phi_s}{\partial z}=0 \quad \text{on } z=\zeta_s,$$

ζ_s is the steady free-surface elevation that satisfies

$$\zeta_s=-\frac{1}{2g}\left(\nabla\Phi_s\cdot\nabla\Phi_s-U^2\right).$$

with $\Phi_s(\mathbf{x})=\Phi_r(\mathbf{x})+\phi(\mathbf{x})$ and $z'=z-\zeta_r$ the free surface condition becomes

$$\begin{aligned}\phi_z+\frac{1}{g}\big[\Phi_{rx}^2\phi_{xx}+2\Phi_{rx}\Phi_{ry}\phi_{xy}+\Phi_{ry}^2\phi_{yy}+\\ (3\Phi_{rx}\Phi_{rxx}+2\Phi_{ry}\Phi_{rxy}+\Phi_{rx}\Phi_{rzz})\phi_x+\\ (3\Phi_{ry}\Phi_{ryy}+2\Phi_{rx}\Phi_{rxy}+\Phi_{ry}\Phi_{rxx})\phi_y\big]=D(x,y) \quad \text{at} \quad z=0,\end{aligned}$$

TU Delft

The steady potential

with

$$D(x,y)=\frac{\partial}{\partial x}[\zeta_r(x,y)\Phi_{rx}(x,y,0)]+\frac{\partial}{\partial y}[\zeta_r(x,y)\Phi_{ry}(x,y,0)]$$

and

$$\zeta_r=\frac{1}{2g}\left[U^2-\Phi_{rx}^2(x,y,0)-\Phi_{ry}^2(x,y,0)\right].$$

The condition advocated by Dawson is

$$\left(\Phi_{rl}^2\phi_l\right)_l+g\phi_z=2\Phi_{rl}^2\Phi_{rll},$$

where l is a curvilinear coordinate along the streamlines of the double body flow.

RAPID (Raven)

Source distribution

$$\Phi_s(\mathbf{x}) = \Phi_\infty(\mathbf{x}) + \int\int_{\partial D} \sigma(\boldsymbol{\xi}) G(\boldsymbol{\xi};\mathbf{x})\, \mathrm{d}S_\xi,$$

where for deep water the Green's function is chosen as

$$G(\boldsymbol{\xi};\mathbf{x}) = -\frac{1}{4\pi r} \qquad r = |\mathbf{x} - \boldsymbol{\xi}|.$$

The integration may be chosen along the ship hull and the actual free surface or raised panels.

Several sources of errors such as

- choice of difference schemes
- position of raised panels $y_{fs} = \alpha \Delta x$
- upstream shift of coolocation points $\gamma \Delta x$
- $\times \quad \times \quad \times$

TU Delft

Dispersion ??

Introduce (notation: Sclavounos and Nakos)

$$s := \frac{k\Delta x}{2\pi} \qquad F_{n\Delta} := \frac{U_\infty}{\sqrt{g\Delta x}}.$$

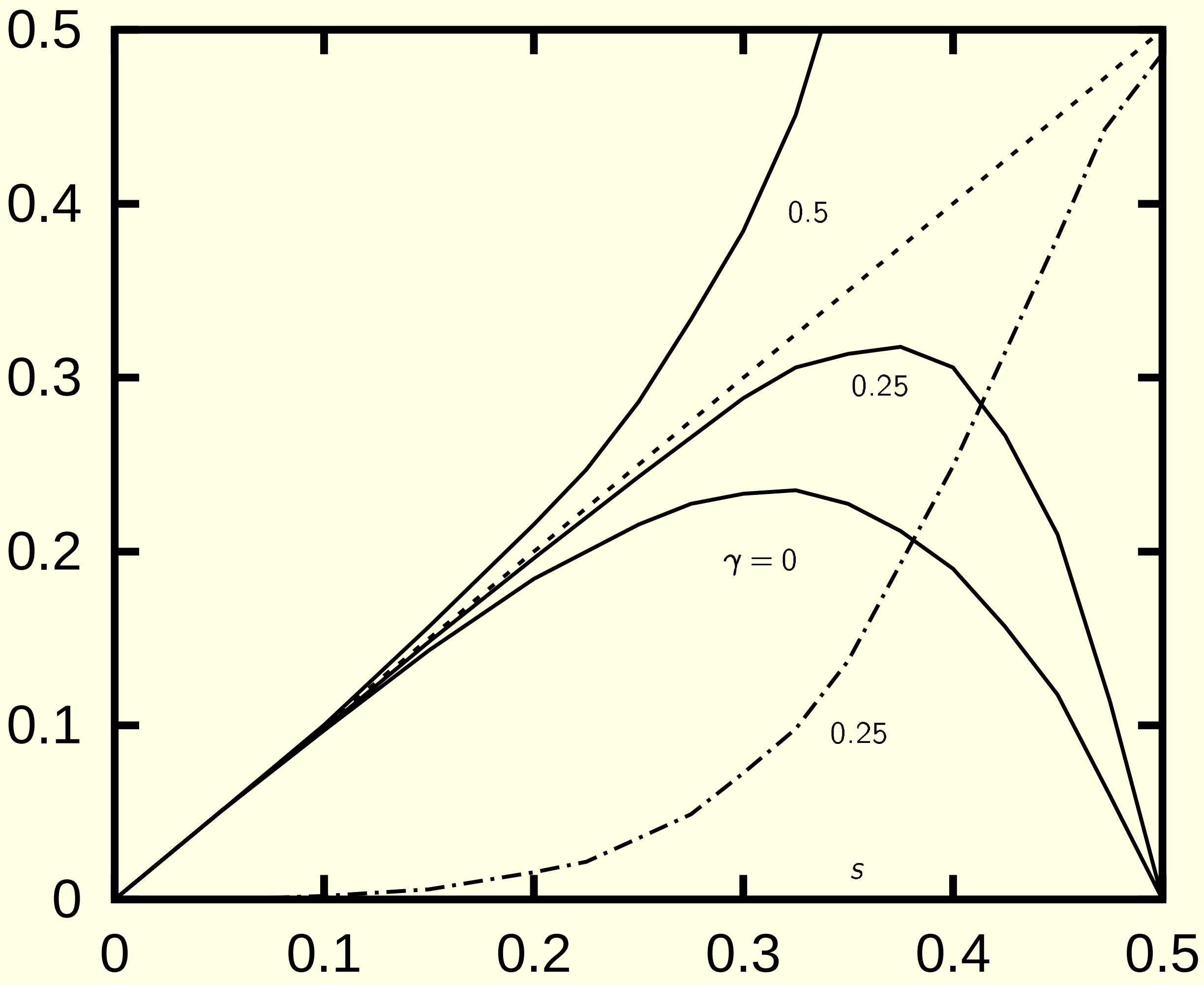
The continuous *dispersion* expression

$$\widetilde{\mathcal{W}}_\nu = k_0 - k = k_0(1 - 2\pi F n_\Delta^2 s).$$

The dispersion relation for the discrete operator becomes

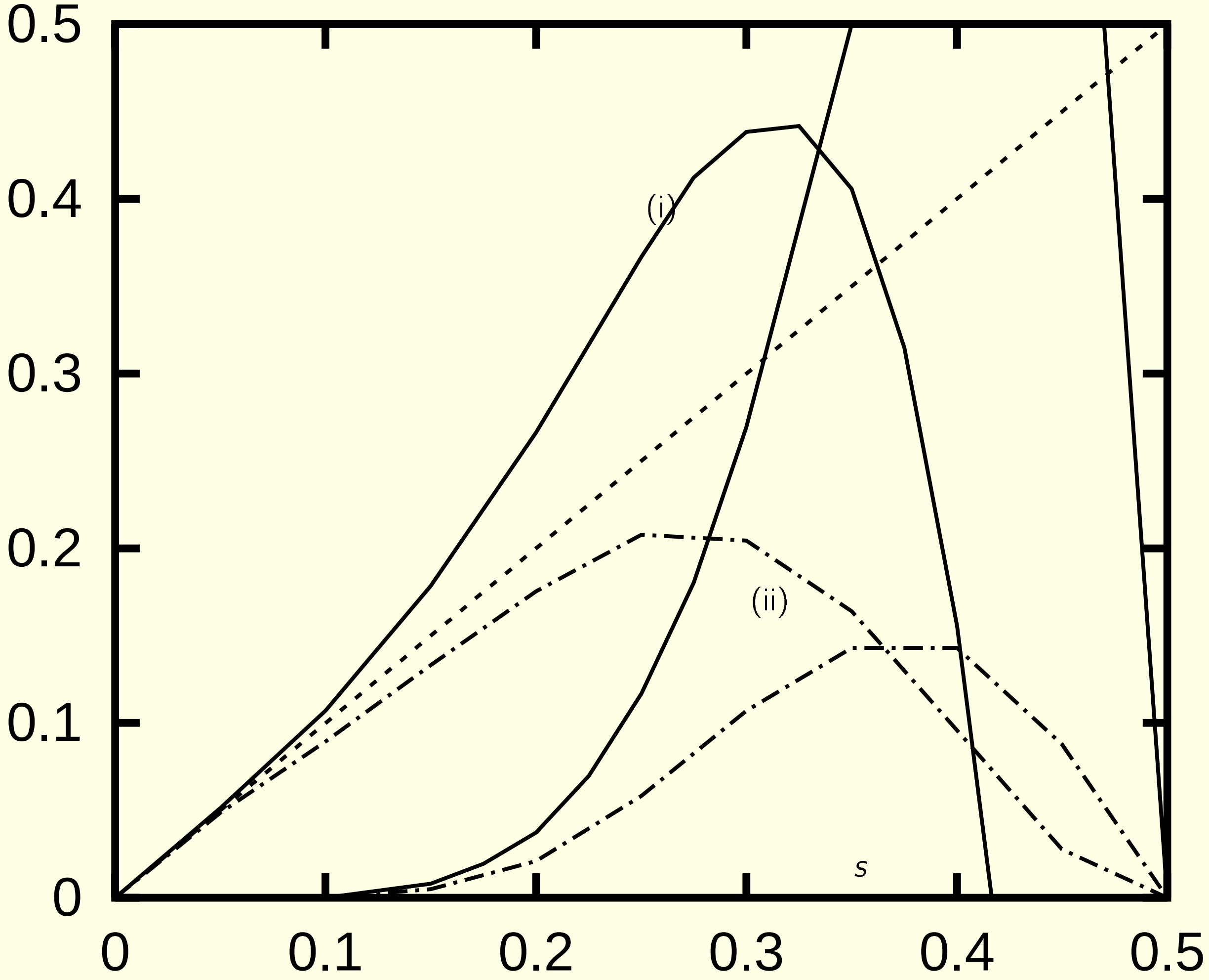
$$\widehat{\mathcal{W}}_\nu = k_0(1 - 2\pi F n_\Delta^2 L h(s)).$$

Deviation in the dispersion



$\Re L h(s)$ for $\alpha = 0.5$ and $\gamma = 0.5,\ 0.25,\ 0$ and $\Im L h(s)$ for $\gamma = 0.25$.

Deviation in the dispersion



$\Re$ and $\Im Lh(s)$ (i) for $\alpha = 1,\ \gamma = 0.25$ and Dawson (ii).

Unsteady potential

We linearise along the steady free surface $z = \zeta_s$

$$\frac{\partial^2 \phi_u}{\partial t^2} + 2\nabla\Phi_s \cdot \nabla \frac{\partial \phi_u}{\partial t} + \nabla\Phi_s \cdot \nabla \left(\nabla\Phi_s \cdot \nabla\phi_u\right) +$$

$$\frac{1}{2}\left(\frac{\partial \phi_u}{\partial x}\frac{\partial}{\partial x_{\zeta_s}} + \frac{\partial \phi_u}{\partial y}\frac{\partial}{\partial y_{\zeta_s}}\right)\|\nabla\Phi_s\|^2 + g\frac{\partial \phi_u}{\partial z} +$$

$$\zeta_u \frac{\partial}{\partial z}\left(\frac{1}{2}\left(\frac{\partial \Phi_s}{\partial x}\frac{\partial}{\partial x_{\zeta_s}} + \frac{\partial \Phi_s}{\partial y}\frac{\partial}{\partial y_{\zeta_s}}\right)\|\nabla\Phi_s\|^2 + g\frac{\partial \Phi_s}{\partial z}\right) = 0.$$

For $\Phi_s = Ux$

$$\frac{\partial^2 \phi_u}{\partial t^2} + 2U\frac{\partial^2 \phi_u}{\partial x \partial t} + U^2\frac{\partial^2 \phi_u}{\partial x^2} + g\frac{\partial \phi_u}{\partial z} = 0 \quad \text{on } z = 0.$$

TUDelft

Slow ship in short waves

For $\Phi_s = \Phi_r(\mathbf{x})$, with $\mathbf{u} = (u, v, w) = \nabla\Phi_r(\mathbf{x})$ (Baba)

$$\frac{1}{g}\left[\frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y}\right]^2 \phi + \frac{\partial}{\partial z}\phi = 0 \quad \text{on } z = 0,$$

Ray expansion

$$\phi(\mathbf{x}, t; k) = a(\mathbf{x}, k)\ \mathrm{e}^{\mathrm{i}kS(\mathbf{x}) - \mathrm{i}\omega t},$$

$$a(\mathbf{x}, k) = \sum_{j=0}^{N} \frac{a_j(\mathbf{x})}{(\mathrm{i}k)^j} + \mathrm{o}((\mathrm{i}k)^{-N}).$$

with $k = \omega^2/g$ and $\omega = \omega_0 + k_0 U \cos\theta$

Asymptotic formulation

Eikonal equation with $\mathbf{p} := (S_x, S_y)$

$$F(x, y, S, p, q) = (1 - \mathbf{u} \cdot \nabla S)^4 - \nabla S \cdot \nabla S = 0.$$

Transport equation

$$\{2\nabla S + 4(1 - \mathbf{u} \cdot \nabla S)^3 \mathbf{u}\} \cdot \nabla a_0 + a_0 M S = 0,$$

where

$$MS = \Delta_3 S - 2\mathbf{u} \cdot \nabla(\mathbf{u} \cdot \nabla S)(1 - \mathbf{u}\nabla S)^2.$$

TUDelft

Characteristic equations

$$\frac{\mathrm{d}x}{\mathrm{d}\sigma} = F_p = -4(1-\mathbf{u}\cdot\mathbf{p})^3 u - 2p,$$

$$\frac{\mathrm{d}y}{\mathrm{d}\sigma} = F_p = -4(1-\mathbf{u}\cdot\mathbf{p})^3 v - 2q,$$

$$\frac{\mathrm{d}p}{\mathrm{d}\sigma} = -(F_x + pF_S) = 4(1-\mathbf{u}\cdot\mathbf{p})^3(\mathbf{u}_x\cdot\mathbf{p}),$$

$$\frac{\mathrm{d}q}{\mathrm{d}\sigma} = -(F_y + qF_S) = 4(1-\mathbf{u}\cdot\mathbf{p})^3(\mathbf{u}_y\cdot\mathbf{p}).$$

and

$$\frac{\mathrm{d}S}{\mathrm{d}\sigma} = pF_p + qF_q = -4(1-\mathbf{u}\cdot\mathbf{p})^3 + 2\mathbf{p}\cdot\mathbf{p}.$$

$$\frac{\mathrm{d}a_0}{\mathrm{d}\sigma} = a_0 MS.$$

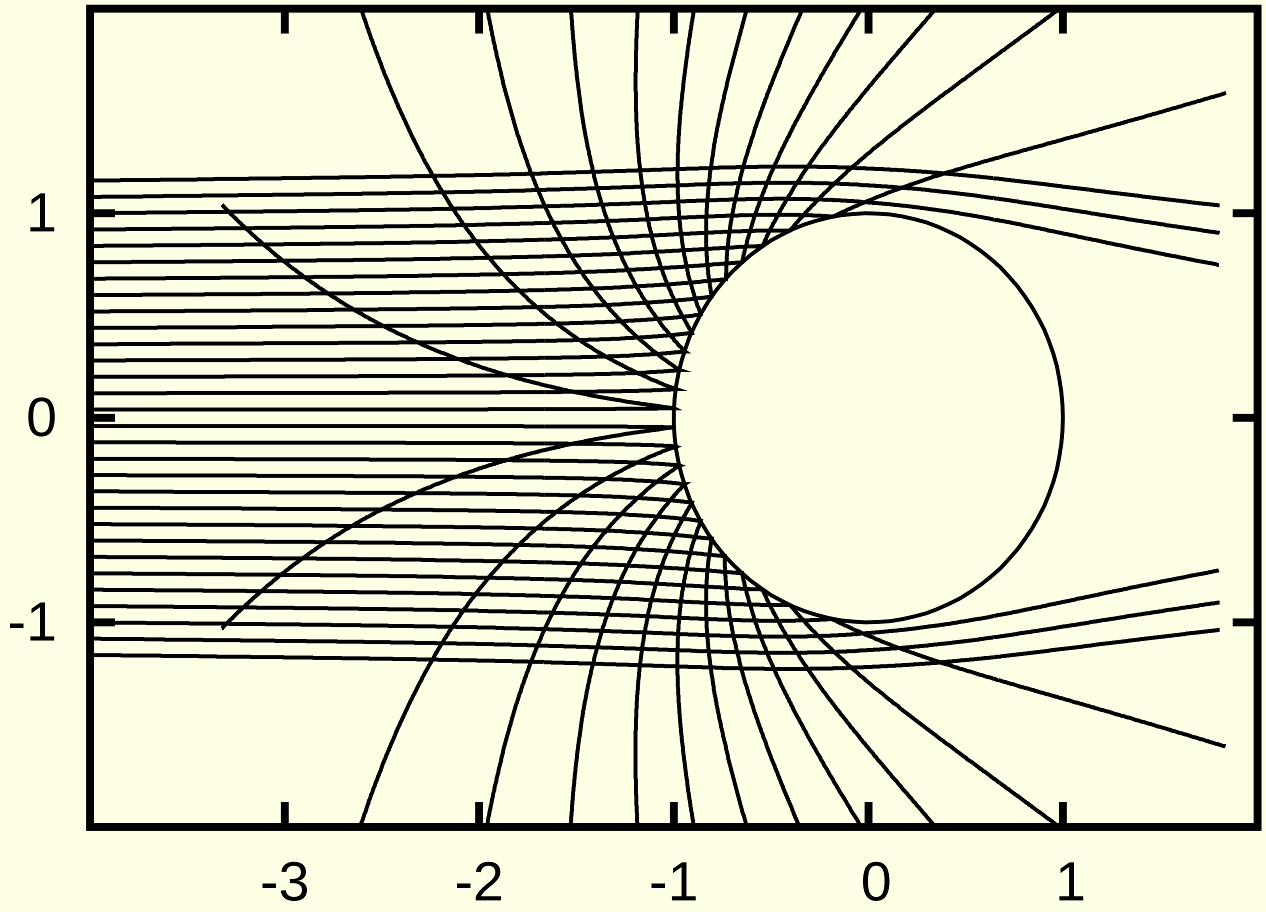
Characteristic equations

This operator MS has the final form

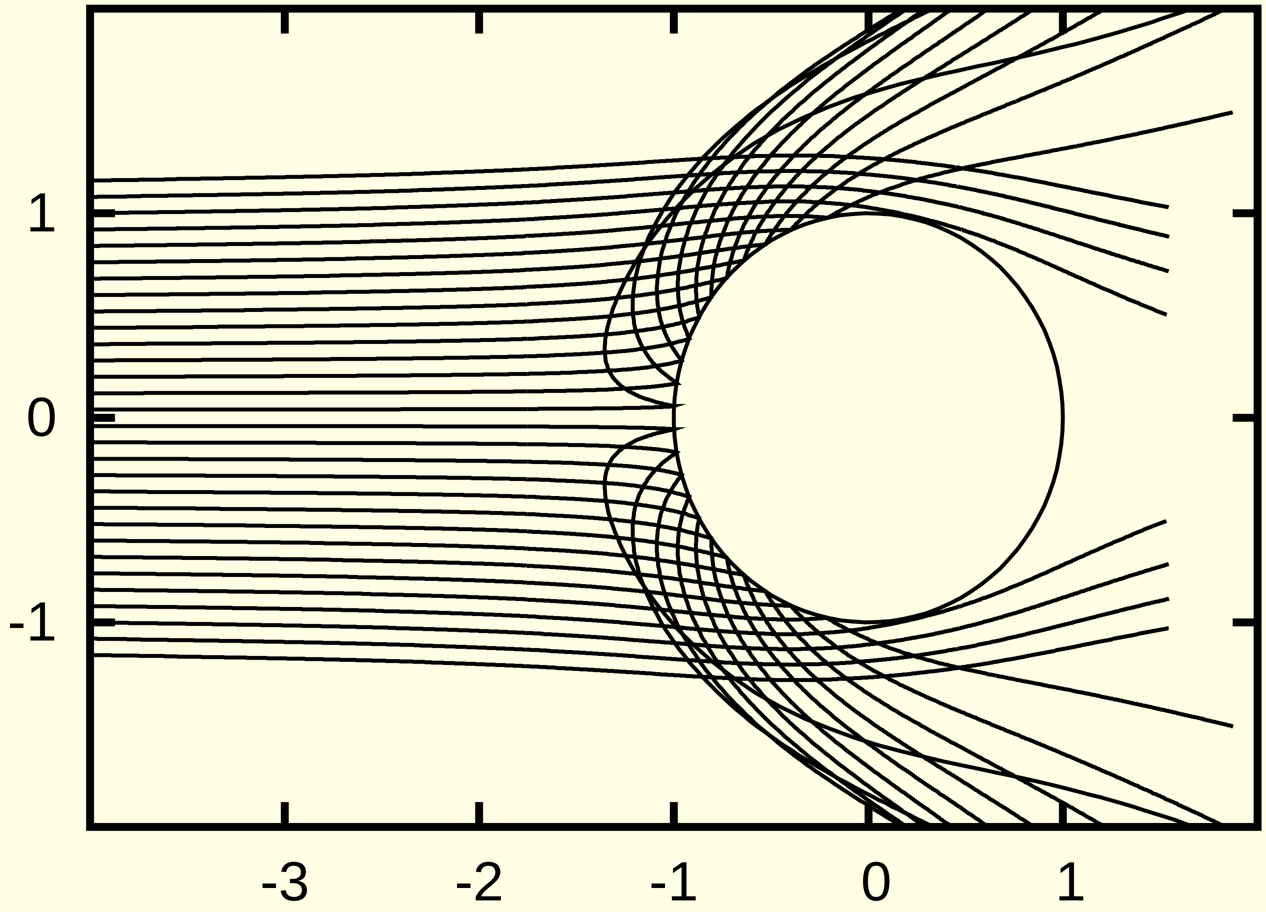
$$
\begin{aligned}
MS = S_{xx}&\left\{1-2|\nabla S|u^2-\frac{S_x^2}{S_x^2+S_y^2}\right\}+\\
S_{xy}&\left\{-4|\nabla S|u^2-2\frac{S_xS_y}{S_x^2+S_y^2}\right\}+\\
S_{yy}&\left\{1-2|\nabla S|u^2-\frac{S_x^2}{S_x^2+S_y^2}\right\}-2|\nabla S|\nabla(\mathbf{u}\cdot\mathbf{u})\cdot\nabla S.
\end{aligned}
$$

TU Delft

Ray pattern $\tau = 0.25$



Ray pattern $\tau = 0.5$



Added resistance

$$\mathbf{F}_{aw} = -\overline{\int_{z=-\infty}^{\zeta} \int_{\mathrm{WL}} p\mathbf{n} \, \mathrm{d}l \, \mathrm{d}z} =$$

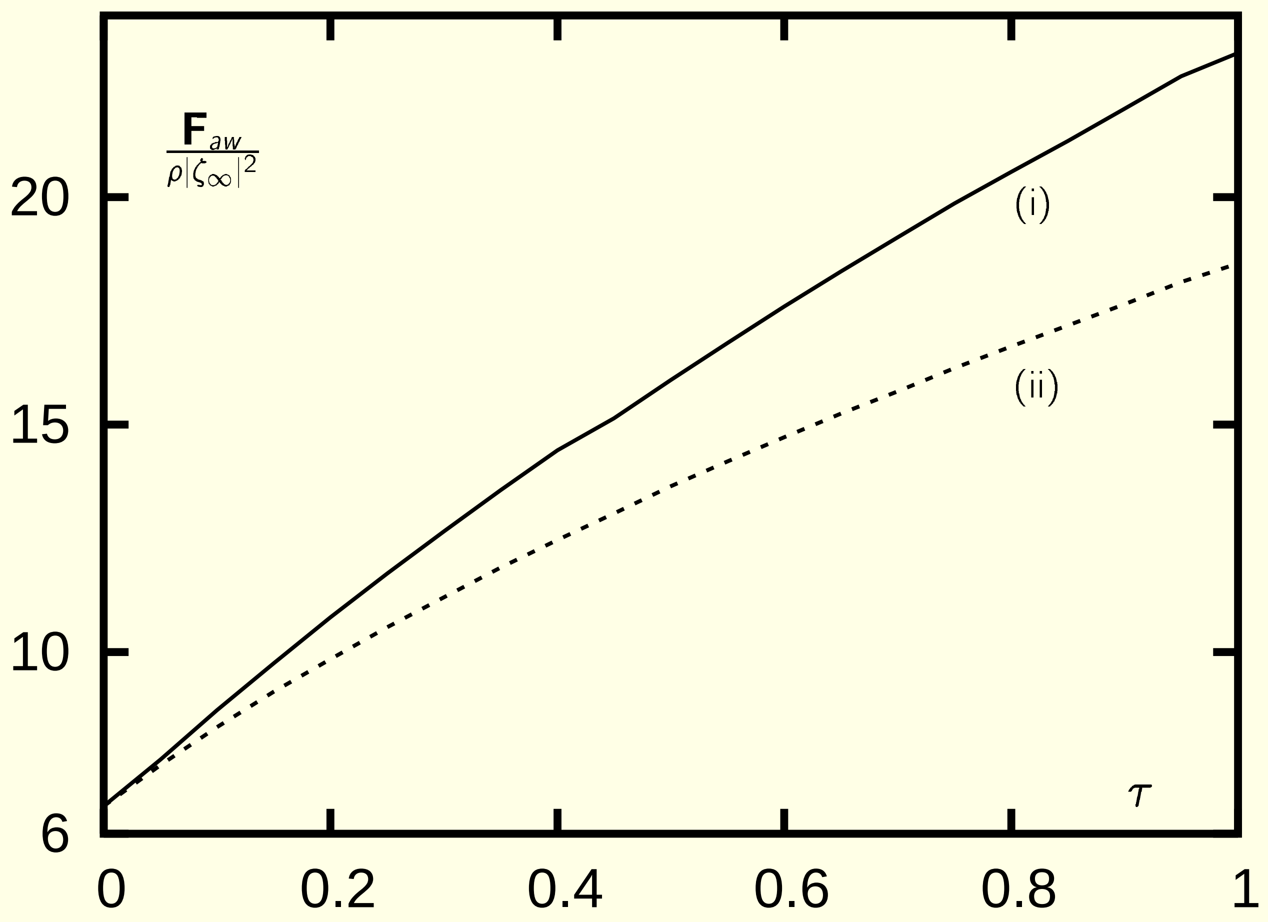
$$-\frac{1}{4}\int_{\mathrm{WL}} \left\{ \left(\nabla S^{(i)} \cdot \nabla S^{(i)}\right)^{\frac{1}{4}} a_0^{(i)} + \left(\nabla S^{(r)} \cdot \nabla S^{(r)}\right)^{\frac{1}{4}} a_0^{(r)} \right\}^2 \mathbf{n} \, \mathrm{d}l$$

$$+\frac{1}{4}\int_{\mathrm{WL}} \left\{ a_0^{(i)^2} \left|\nabla S^{(i)}\right| + a_0^{(r)^2} \left|\nabla S^{(r)}\right| + \right.$$

$$\left. 2a_0^{(i)} a_0^{(r)} \frac{\nabla S^{(i)} \cdot \nabla S^{(r)} + \left|\nabla S^{(i)}\right| \left|\nabla S^{(r)}\right|}{\left|\nabla S^{(i)}\right| + \left|\nabla S^{(r)}\right|} \right\} \mathbf{n} \, \mathrm{d}l.$$

TUDelft

Added resistance for (i) a circular cylinder and (ii) a sphere



TU Delft

Numerical formulation

With time-step Δt we use the second order difference schemes:

$$\frac{\partial^2 \phi^i}{\partial t^2} = \frac{1}{(\Delta t)^2}\left(2\phi^i - 5\phi^{i-1} + 4\phi^{i-2} - \phi^{i-3}\right) + \mathcal{O}\left((\Delta t)^2\right)$$

$$\frac{\partial \phi^i}{\partial t} = \frac{1}{\Delta t}\left(\frac{3}{2}\phi^i - 2\phi^{i-1} + \frac{1}{2}\phi^{i-2}\right) + \mathcal{O}\left((\Delta t)^2\right)$$

This leads to the following FSC

TU Delft

Numerical formulation

$$\phi^i\left(\frac{2}{(\Delta t)^2}-\frac{T}{gS}\frac{3}{2\Delta t}\right)+\nabla\Phi_s\cdot\nabla\left(\nabla\Phi_s\cdot\nabla\phi^i\right)+$$

$$\left(2\frac{3}{2\Delta t}-\frac{T}{gS}\right)\nabla\phi^i\cdot\nabla\Phi_s+g\frac{\partial\phi^i}{\partial z}+$$

$$\frac{1}{2}\left(\frac{\partial\phi^i}{\partial x}\frac{\partial}{\partial x_{\zeta_s}}+\frac{\partial\phi^i}{\partial y}\frac{\partial}{\partial y_{\zeta_s}}\right)\|\nabla\Phi_s\|^2=f \quad \text{on } z=\zeta_s$$

$$T=\frac{\partial}{\partial z}\left(\frac{1}{2}\left(\frac{\partial\Phi_s}{\partial x}\frac{\partial}{\partial x_{\zeta}}+\frac{\partial\Phi_s}{\partial y}\frac{\partial}{\partial y_{\zeta}}\right)\|\nabla\Phi_s\|^2+g\frac{\partial\Phi_s}{\partial z}\right) \quad \text{on } z=\zeta_s$$

$$S=1+\frac{1}{2g}\frac{\partial}{\partial z}\|\nabla\Phi_s\|^2$$

$$f=\frac{5\phi^{i-1}-4\phi^{i-2}+\phi^{i-3}}{(\Delta t)^2}+\left(2\nabla\Phi_s\cdot\nabla-\frac{T}{gS}\right)\frac{\left(2\phi^{i-1}-\frac{1}{2}\phi^{i-2}\right)}{\Delta t}$$

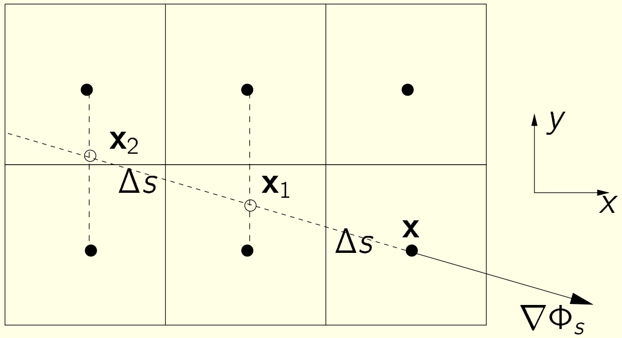
TU Delft

Numerical formulation

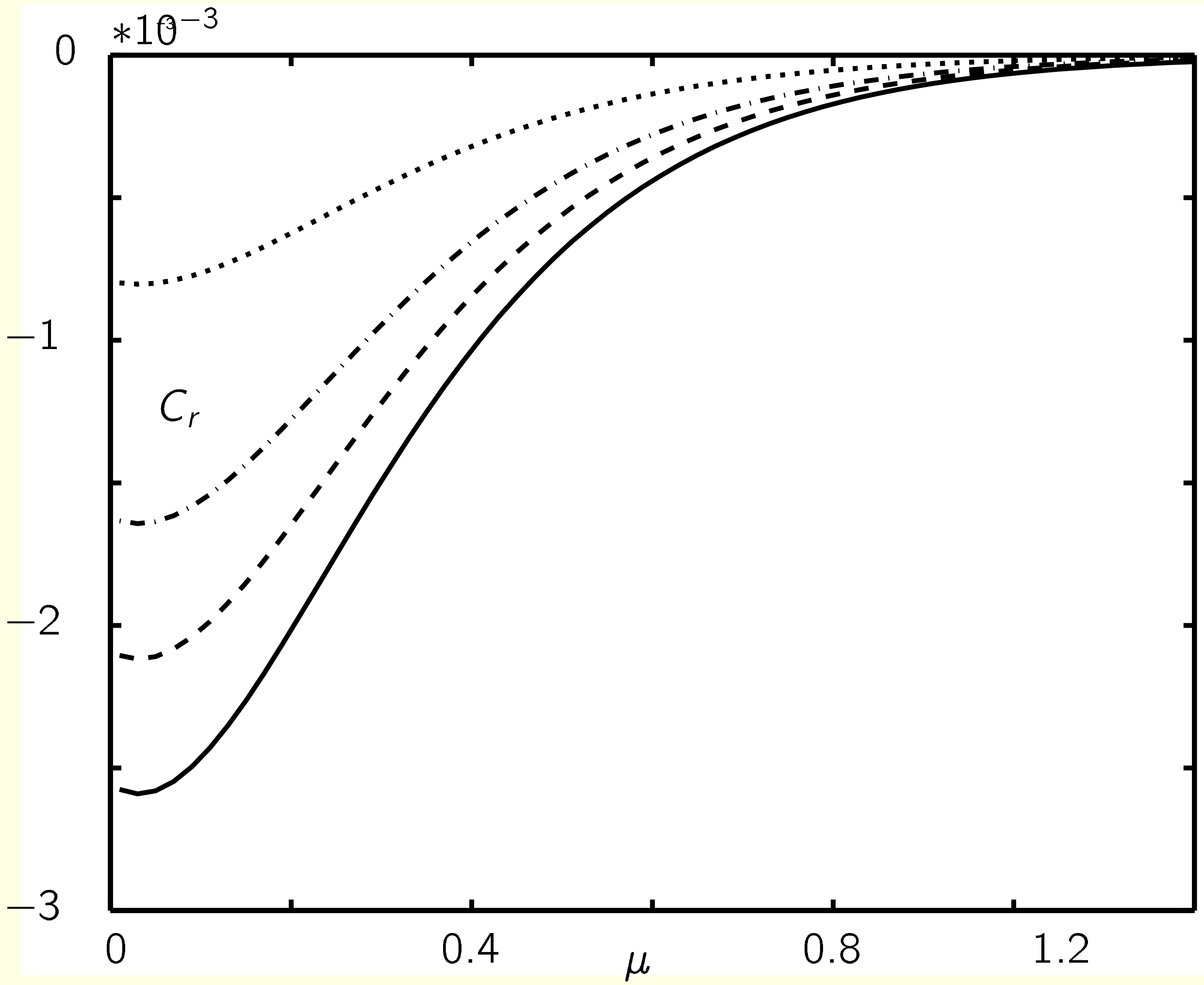
For example, we write

$$\nabla\Phi_s\cdot\nabla\phi = \|\nabla\Phi_s\|\frac{\frac{3}{2}\phi(\mathbf{x}) - 2\phi(\mathbf{x}_1) + \frac{1}{2}\phi(\mathbf{x}_2)}{\Delta s} + \mathcal{O}\left((\Delta s)^2\right)$$

difference scheme for $\nabla\Phi_s\cdot\nabla\phi$.

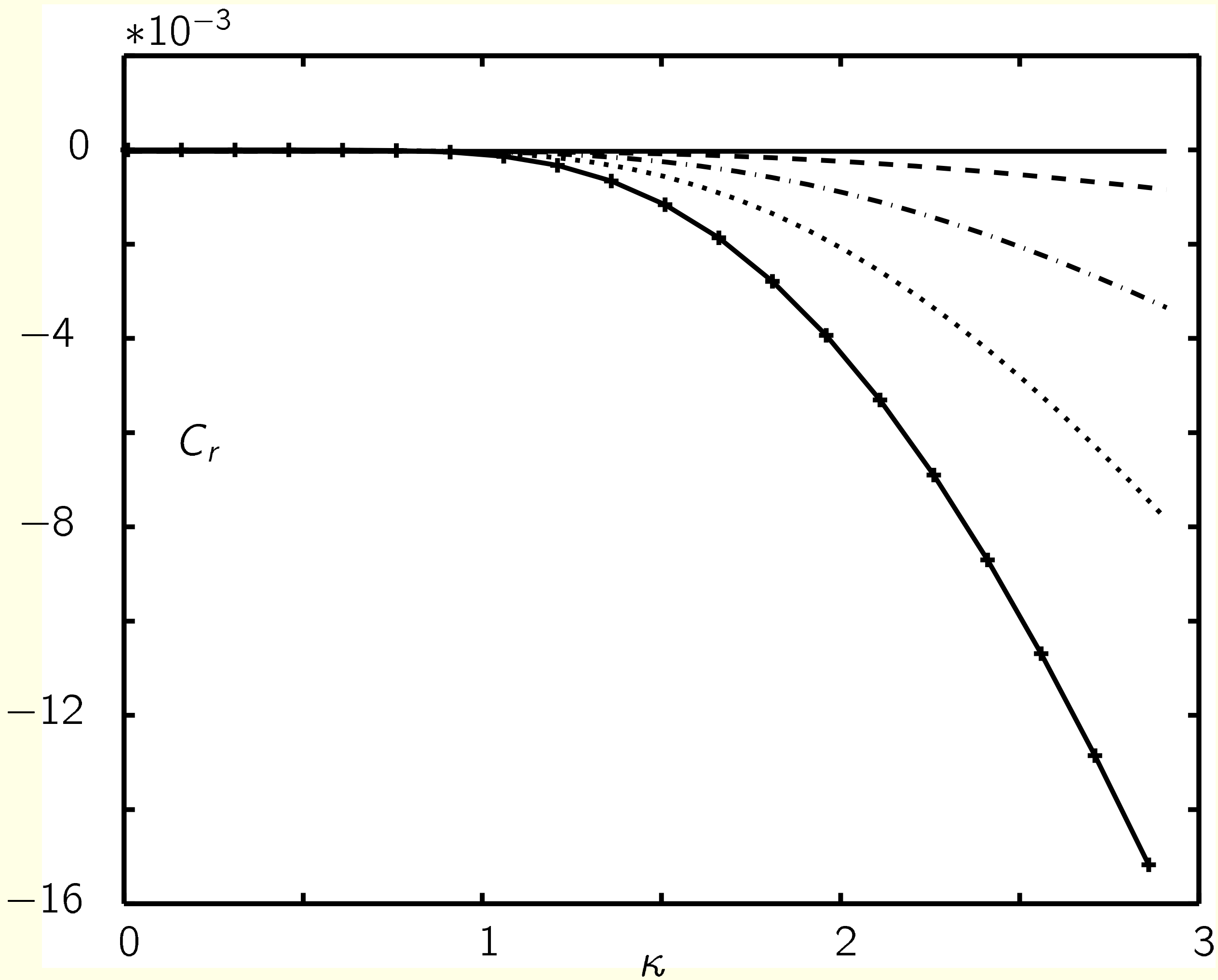


Deviation in the dispersion



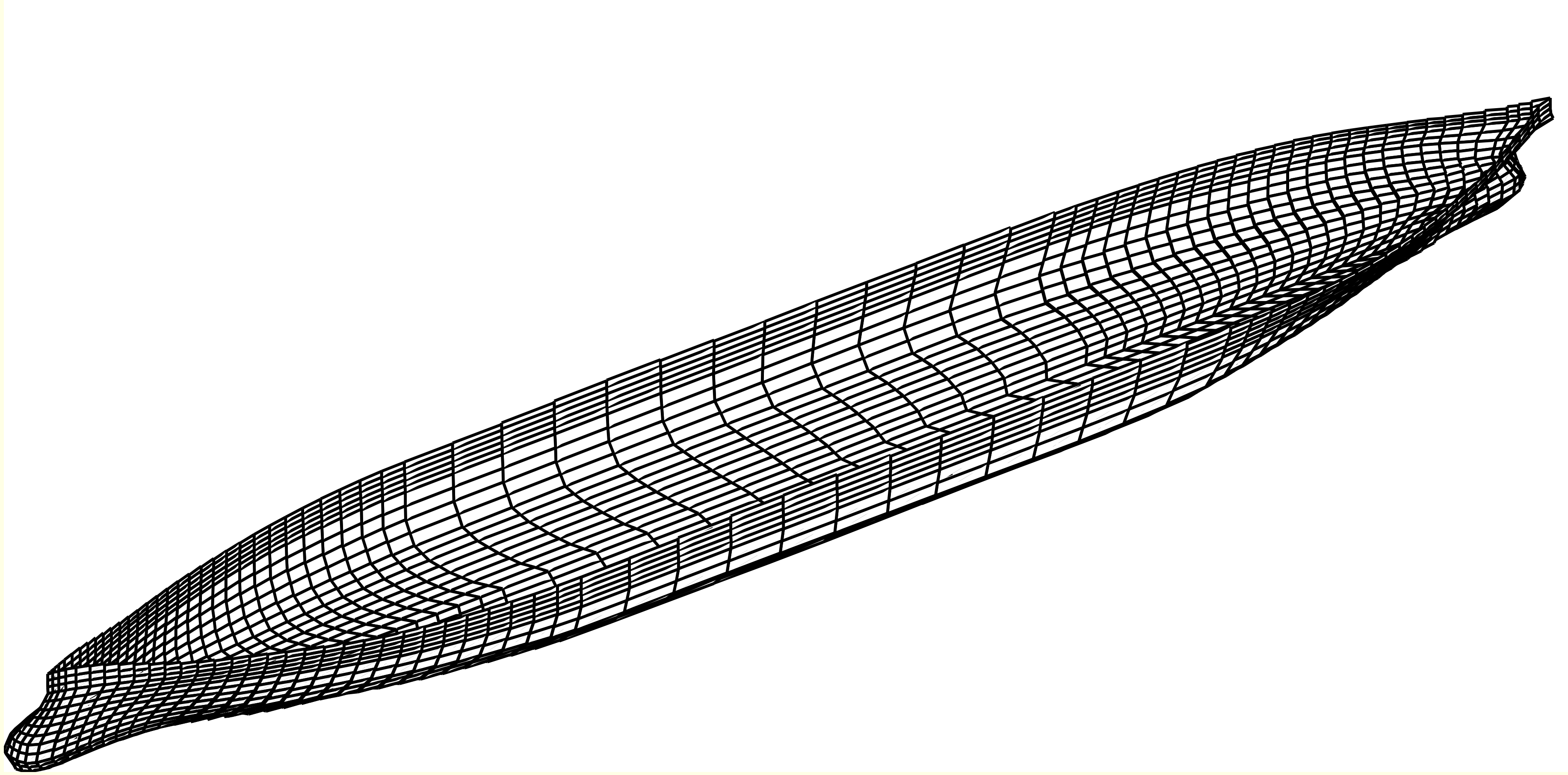
$\tau = 5,\ 1,\ 0.5,\ 0.25$ (top down), $Fn = 0.4$, $\theta = 0$, $\kappa = 1$, $\widehat{\alpha} = 0.05$.

Deviation in the dispersion

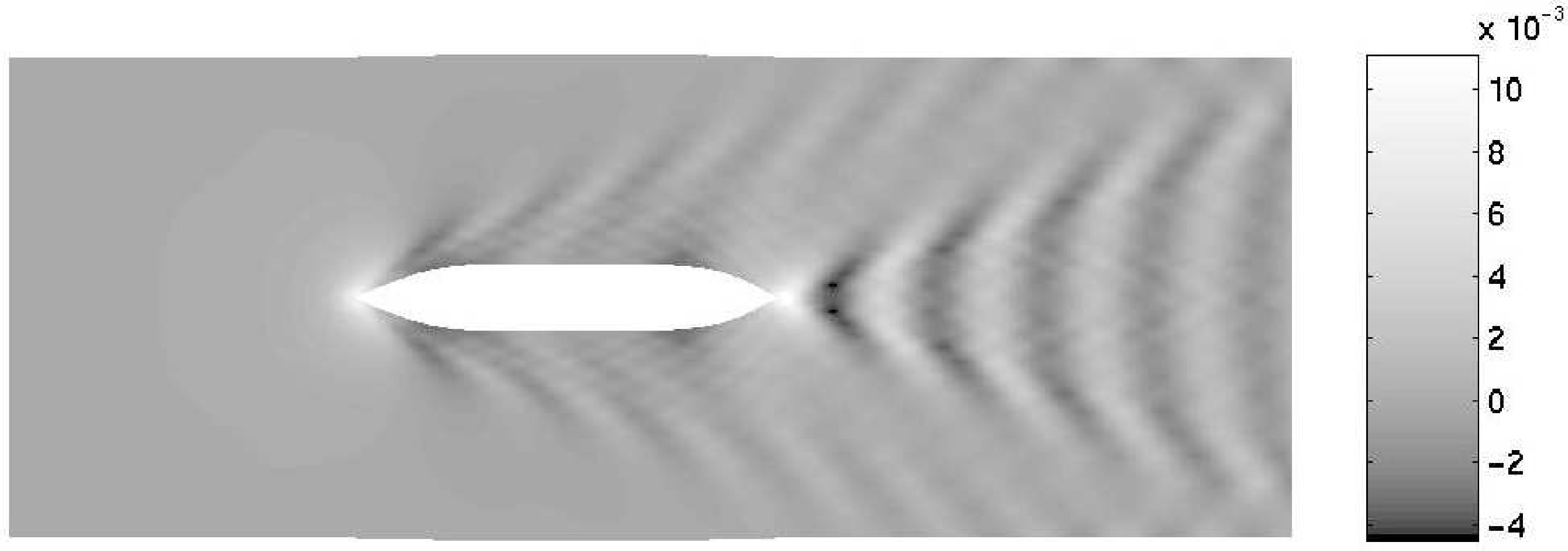


$\theta = 0,\ \pi/8,\ \pi/4,\ 3\pi/4,\ \pi/2$ (top down), $Fn = 0.4$, $\widehat{\alpha} = 0.05$, $\mu = 1$, $\lambda = 1$.

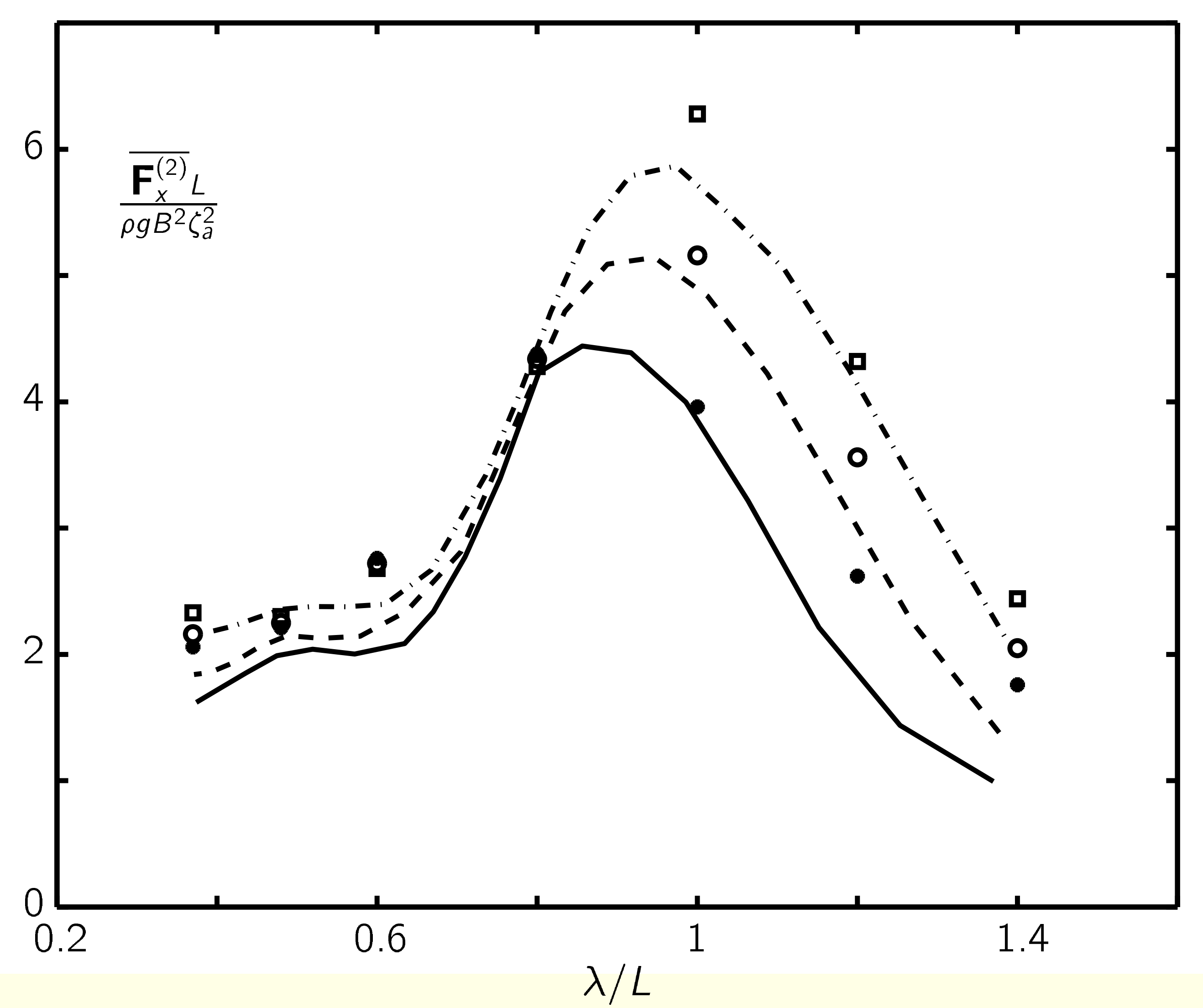
Hull paneling of LNG carrier for Froude number 0.2.



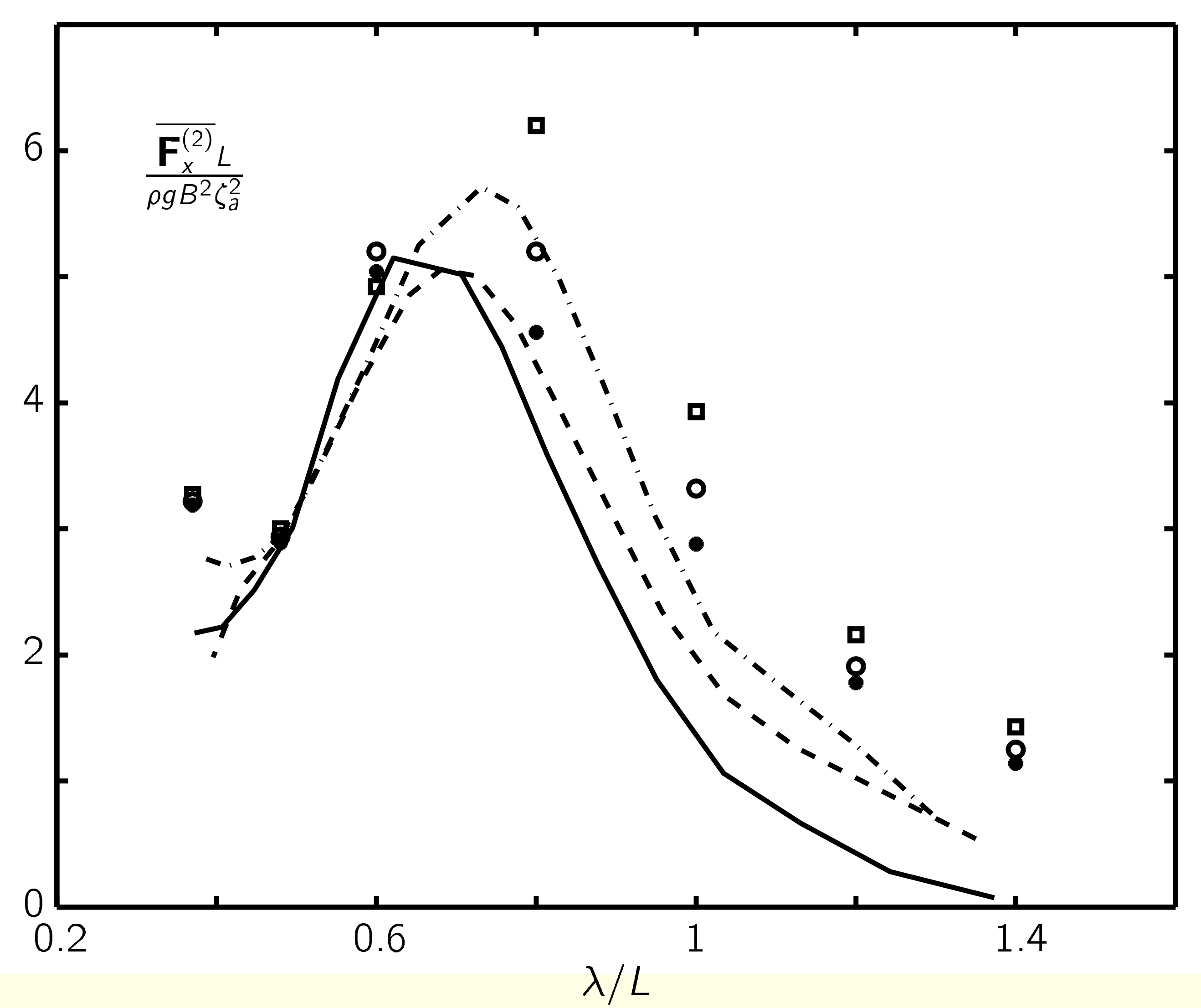
Steady wave pattern for Froude number 0.2



Added resistance $F_n = 0.2, 0.17, 0.14$

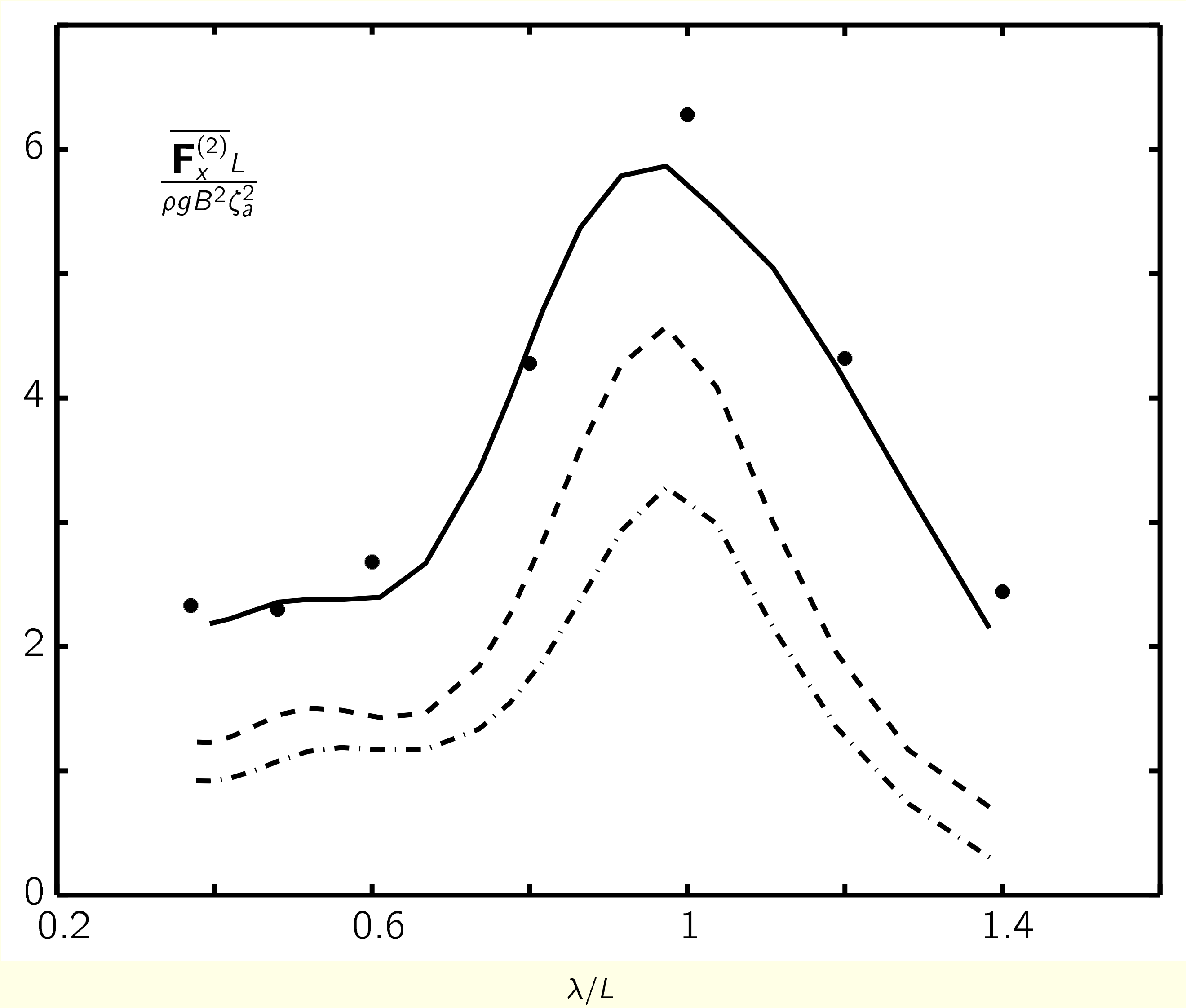


Added resistance (bow-quartering)

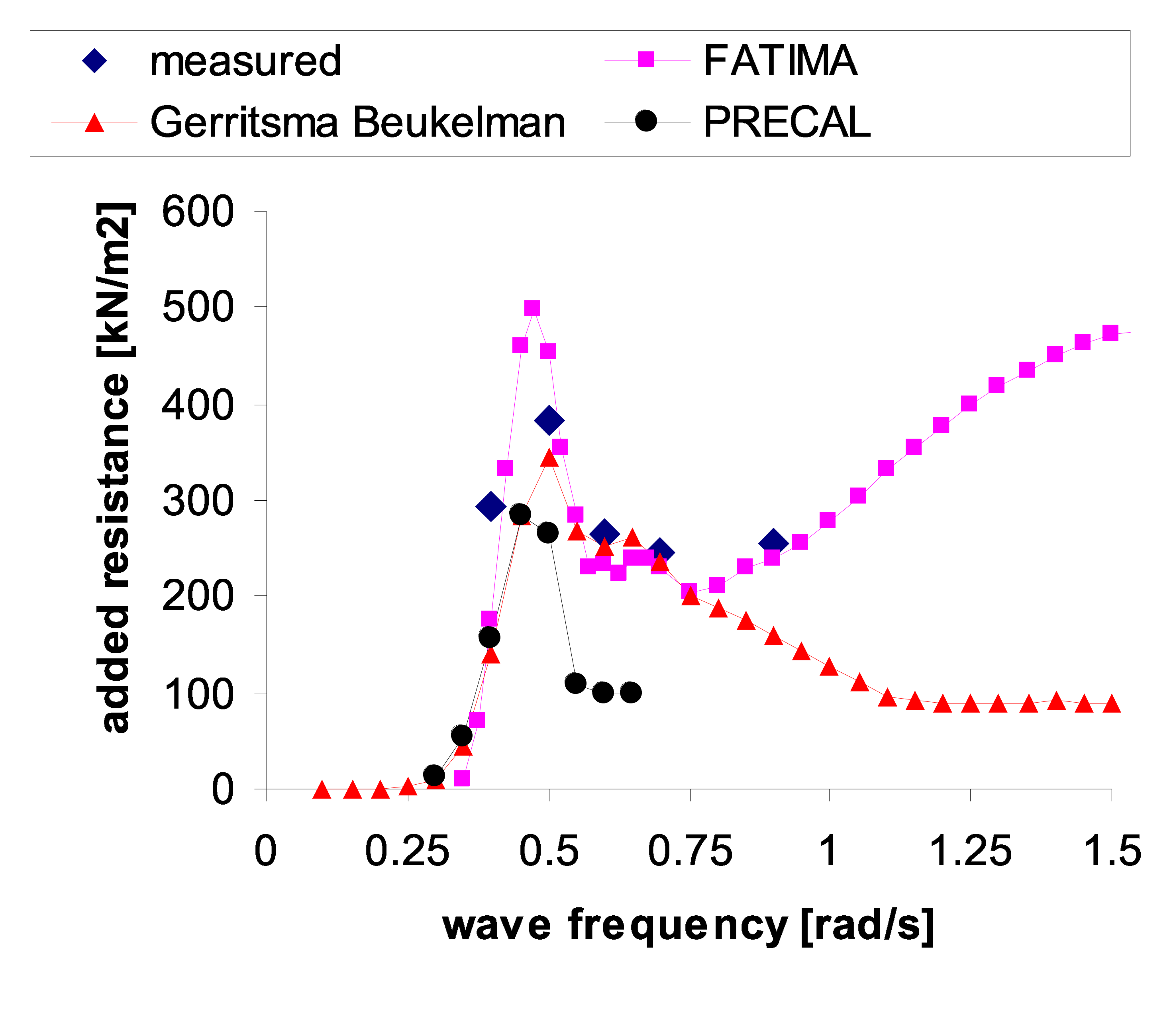


TUDelft

Added resistance with different steady flow



Results cruise carrier



Comparison with asymptotic results