The Effect of Moderate Speed on the Motion of Floating Bodies

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1. INTRODUCTION

It is an honour for me to address the audience at this Festkolloquium in honour of the retirement of Professor Dr. Klaus Eggers. I will discuss a problem I am working on together with my co-author Huijsmans of MARIN. It is often difficult to trace the point where one gets the idea to tackle a problem in a specific way. A discussion with Prof. Eggers on a different topic, namely the treatment of the free surface condition in the low Froude number wave resistance problem convinced me that a careful consideration of the free surface condition in the ship motion problem could lead to a proper theory just as well.
In this presentation I will show how in the case of a ship with moderate forward speed the ship motion can be dealt with. In the case of a slender ship this has been shown in [9].

In 1971 Remery and Hermans [19] reported results of the excitation and motion of a barge moored to a single point in wave groups. They considered the surge motion only and showed that excitation of the large amplitude low frequency motion was due to the low frequency drift force. At the time they used in their simplified model a non realistic large damping coefficient to predict the low frequency motion response. In 1980 Pinkster [18] published a method to compute the low frequency drift force. Pressure integration techniques resulted in excellent agreement of the calculated drift forces at zero forward speed with experiments. An unsolved problem, however, is the estimation of the motion of a moored ship, especially when the mooring is unstable. Wichers and Huijsmans [23] showed that the damping at the natural frequencies of the mooring system have to be considered carefully. Results of model test experiments showed that a large part of the damping at these natural frequencies could be generated by the velocity dependency of the wave drift forces. Hence, the effect of moderate speed should be accounted for if one wants to evaluate this wave damping phenomenon.

In the last decade many theories have been developed to compute ship motions.

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Major attention is directed towards thin and slender ships sailing at forward speed in waves. An important development is the slender body approximation by Newman [13] and Newman and Tuck [14] for both zero and non-zero forward speed. The unified strip theory introduced by Newman [16] produced again better results in several cases and is extended by several authors. Meanwhile, some powerful programs have been introduced to treat the zero speed case for ship motions, without any geometrical simplifications. They are based on the solution of integral equations, where the potential function is written as a source distribution or by means of Green's theorem. An improved treatment of the pulsating source term is given by Newman [16] and Noblesse [20].

At MARIN the original diffraction program of Van Oortmerssen [17] is updated with this new procedure. Model tests with several exceptional geometries show that this program is very reliable. There is no program available with the same performance in the case of forward speed. A direct approach as reported by Bougis [2], Chang [4] and Inglis [11] is feasible, however, it is time consuming. An other defect of that approach is the improper treatment of the free surface. Without mentioning it a slenderness property is assumed to neglect the effect of the disturbance of the stationary part of the potential function. For these reasons we derive a consistent method based on a perturbation with respect to the small value of the Froude number.

2. MATHEMATICAL FORMULATION

The total potential function will be split in a steady and a nonsteady part in a well-known way:

$$\Phi(\underline{x},t) = Ux + \bar{\Phi}(\underline{x};U) + \tilde{\Phi}(\underline{x},t;U) \tag{1}$$

in this formulation U is the incoming unperturbed velocity field, obtained by considering a coordinate system fixed to a ship moving under a drift angle α. In our approach this angle need not be small. The time dependent part of the potential consists of an incoming wave at frequency ω, a diffracted and/or a radiated wave contribution. To compute the drift force all these components will be taken into account. In this paper we restrict ourselves to a general theory concerning the wave components. Some results of drift force computations are shown as well.

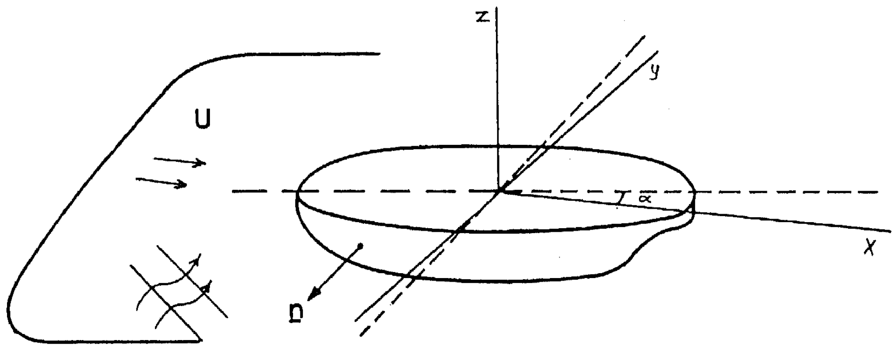

Fig. 1.

The equations for the total potential Φ can be written as:

$$\Delta\Phi = 0 \quad \text{in the fluid domain } D_e \tag{2}$$

At the free surface we have the dynamic and kinematic boundary condition.

$$\left.\begin{aligned} & g\zeta + \Phi_t + \frac{1}{2}\nabla\Phi.\nabla\Phi = \text{const.} \\ & \Phi_z - \Phi_x\zeta_y - \Phi_y\zeta_y - \zeta_t = 0 \end{aligned}\right\} \text{ at } z = \zeta(x,y,t) \tag{3}$$

We assume that the waves are high compared with the Kelvin wave pattern and that they both are small, hence the free surface condition can be expanded at z = 0. Elimination of ζ leads to the following nonlinear condition:

$$\frac{\partial^2\Phi}{\partial t^2} + g\frac{\partial\Phi}{\partial z} + \frac{\partial}{\partial t}(\nabla\Phi.\nabla\Phi) + \nabla\Phi.\nabla\left(\frac{\nabla\Phi.\nabla\Phi}{2}\right) = 0 \quad \text{at } z = 0 \tag{4}$$

To compute the wave resistance at low speed the free surface must be treated more carefully, because the wave height is of asymptotically smaller order. This problem is studied extensively by Eggers [5], Baba [1], Hermans [8] and Brandsma [3]. The velocity field is well described by the double body potential with a small wave pattern. Therefore we take the double body potential into account and we neglect the stationary wave pattern.

For the wave potential $\tilde{\phi}(\underline{x},t;U)$ the free surface condition now becomes:

$$\begin{aligned}
&\tilde{\phi}_{tt} + g\tilde{\phi}_z + 2U\tilde{\phi}_{xt} + 2\nabla\bar{\Phi}.\nabla\tilde{\phi}_{\ t} + \\
&+ (U^2 + 2U\bar{\Phi}_x + \bar{\Phi}_x^2)\tilde{\phi}_{xx} + 2(U + \bar{\Phi}_x)\bar{\Phi}_y\tilde{\phi}_{xy} + \\
&+ \bar{\Phi}_y^2\tilde{\phi}_{yy} + (3U\bar{\Phi}_{xx} + \bar{\Phi}_x\bar{\Phi}_{xx} + \bar{\Phi}_y\bar{\Phi}_{xy})\tilde{\phi}_x + \\
&+ (2U\bar{\Phi}_{xy} + \bar{\Phi}_x\bar{\Phi}_{xy} + \bar{\Phi}_y\bar{\Phi}_{yy})\tilde{\phi}_y + L^{(2)}\{\tilde{\phi}\} = 0 \qquad \text{at } z = 0
\end{aligned} \tag{5}$$

The boundary condition on the hull can be written in a similar way for all radiating and diffracted modes. We therefore treat the following general form, keeping in mind that the actual form has to be used in the computations. Generally we have the condition:

$$(\nabla\tilde{\phi}.\underline{n}) = V(\underline{x})e^{-i\omega t} \quad \text{at the mean position of the hull } \underline{x} \in S \tag{6}$$

The nonlinear operator on $\tilde{\phi}$ will be neglected as well. The first line in (5) contains linear terms in U. Our Ansatz is that in order to obtain the first order approximation with respect to U the second order terms with respect to U may be neglected in the free surface. In the next section we show that in general this is true, but first we discuss the construction of the regular part of the perturbation problem, with the complete linear free surface condition.

We assume $\tilde{\phi}(\underline{x},t;U)$ to be oscillatory:

$$\tilde{\phi}(\underline{x},t;U) = \phi(\underline{x},U)e^{-i\omega t} \tag{7}$$

The free surface condition is written as:

$$-\omega^2\phi - 2i\omega U\phi_x + U^2\phi_{xx} + g\phi_z = D(U;\bar{\Phi})\{\phi\} \quad \text{at } z = 0 \tag{8}$$

where $D(U;\bar{\Phi})$ is a linear differential operator acting on ϕ as defined in (5). We apply Green's theorem to a problem in D_i inside S and to the problem in D_e outside S where S is the ship's hull. The potential function inside S obeys condition (8) with D = 0, while the Green's function fulfils the homogeneous adjoint free surface condition:

$$- \omega^2 G + 2i\omega U G_\xi + U^2 G_{\xi\xi} + g G_\zeta = 0 \qquad \text{at } \zeta = 0 \tag{9}$$

This Green's function has the form:

$$G(\underline{x}, \underline{\xi}; U) = -\frac{1}{r} + \frac{1}{r_1} - \psi(\underline{x}, \underline{\xi}; U) \tag{10}$$

where $r = |\underline{x} - \underline{\xi}|$ and $r_1 = |\underline{x} - \underline{\xi}'|$, where $\underline{\xi}'$ is the image of $\underline{\xi}$ with respect to the free surface.

Combining the formulation inside and outside the ship we may obtain a description of the potential function defined outside S by means of a source and vortex distribution of the following form:

$$\begin{aligned} &- \iint_S \gamma(\underline{\xi}) \frac{\partial}{\partial n} G(\underline{x},\underline{\xi}) dS_\xi - \iint_S \sigma(\underline{\xi}) G(\underline{x},\underline{\xi}) dS_\xi - \frac{2i\omega U}{g} \int_{WL} \gamma(\underline{\xi}) G(\underline{x},\underline{\xi}) d\eta \\ &+ \frac{U^2}{g} \int_{WL} [\gamma(\underline{\xi}) \frac{\partial}{\partial \xi} G(\underline{x},\underline{\xi}) - \{\alpha_t \gamma_t(\underline{\xi}) + \alpha_T \gamma_T(\underline{\xi})\} G(\underline{x},\underline{\xi})] d\eta \\ &+ \frac{U^2}{g} \int_{WL} \alpha_n \sigma(\underline{\xi}) G(\underline{x},\underline{\xi}) d\eta + \frac{i\omega}{g} \iint_{FS} G(\underline{x},\underline{\xi}) D\{\phi\} dS_\xi = 4\pi\phi(\underline{x}) \end{aligned} \tag{11}$$

$\alpha_t = \cos(0x,\underline{t})$, $\alpha_T = \cos(0x,\underline{T})$, $\alpha_n = \cos(0x,\underline{n})$

where n is the normal and t the tangent to the waterline and $T = t \times n$ the binormal.

It is clear that with the choice $\gamma(\underline{\xi}) = 0$ the integral along the waterline gives no contribution up to order U. The source distribution we obtain this way is not a proper distribution, because it expresses the function ϕ in a source distribution along the free surface with a strength proportional to derivatives of the same function ϕ. However, this formulation is linear in U and moreover the integrand tends to zero rapidly for increasing distance R. So finally we arrive at the formulation:

$$\begin{aligned} &- 2\pi\sigma(\underline{x}) - \iint_S \sigma(\underline{\xi}) \frac{\partial G}{\partial n_x}(\underline{x},\underline{\xi}) dS_\xi + \frac{U^2}{g} \int_{WL} \alpha_n \sigma(\underline{\xi}) \frac{\partial G}{\partial n_x}(\underline{x},\underline{\xi}) d\eta \\ &+ \frac{i\omega}{g} \iint_{FS} \frac{\partial}{\partial n_x} G(\underline{x},\underline{\xi}) D\{\phi\} dS_\xi = 4\pi V(\underline{x}), \quad \underline{x} \in S \end{aligned} \tag{12}$$

and

$$4\pi\phi(\underline{x}) = -\iint_S \sigma(\underline{\xi})G(\underline{x},\underline{\xi})dS_\xi + \frac{U^2}{g}\int_{WL}\alpha_n\sigma(\underline{\xi})G(\underline{x},\underline{\xi})d\eta$$
$$+ \frac{i\omega}{g}\iint_{FS} G(\underline{x},\underline{\xi})D\{\phi\}dS_\xi \qquad \underline{x} \in D_e \tag{13}$$

We now consider small values of U, keeping in mind that there are two dimensionless parameters that play a role in the limit. We consider $\tau = \frac{U\omega}{g} << 1$ and $\nu = \frac{gL}{U^2} >> 1$. It turns out that the source strength and the potential function can be expanded as follows:

$$\sigma(\underline{x};U) = \sigma_0(x) + \tau\sigma_1(\underline{x}) + \tilde{\sigma}(\underline{x};U)$$
$$\phi(\underline{x};U) = \phi_0(x) + \tau\phi_1(x) + \tilde{\phi}(x;U) \tag{14}$$

where $\tilde{\sigma}$ and $\tilde{\phi}$ are $O(\tau^2)$ as $\tau \to 0$, while the expansion of the Green's function is less trivial.

3. THE GREEN'S FUNCTION

In this section we present an asymptotic expansion of the Green's function. The Green's function follows from the source function presented in Wehausen and Laitone [22].

In the case $\tau < 1/4$ the function $\psi(\underline{x},\underline{\xi};U)$ is written as follows:

$$\psi(\underline{x},\underline{\xi};U) = \frac{2g}{\pi}\int_0^{\pi/2} d\theta \int_{L_1} dk\ F(\theta,k) + \frac{2g}{\pi}\int_{\pi/2}^{\pi} d\theta \int_{L_2} dk\ F(\theta,k) \tag{15}$$

where:

$$F(\theta,k) = \frac{k\exp(k[z+\zeta+i(x-\xi)\cos\theta])\cos[k(y-\eta)\sin\theta]}{gk - (\omega + kU\cos\theta)^2} \tag{16}$$

The contours L_1 and L_2 are given as follows:

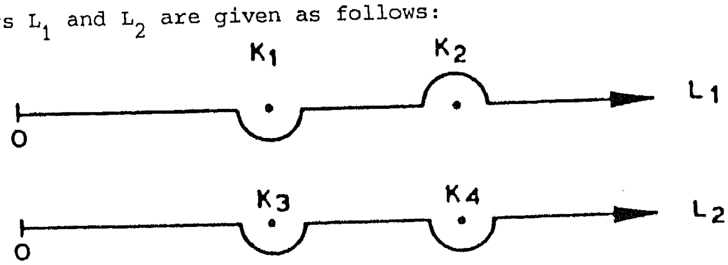

Fig. 2.

These contours are chosen such that the 'radiation' conditions are satisfied. The radiated waves are outgoing and the Kelvin pattern is behind the ship. The values k are the poles of $F(\theta,k)$. For small values of τ these poles behave as follows:

$$\sqrt{gk_1},\quad \sqrt{gk_3} \sim \omega + O(\tau) \qquad \text{as } \tau \to 0 \tag{17}$$

$$\sqrt{gk_2},\quad -\sqrt{gk_4} \sim \frac{\omega}{\tau\cos\theta} + O(1) \qquad \text{as } \tau \to 0 \tag{18}$$

A careful analysis of the asymptotic behaviour of $\psi(\underline{x},\underline{\xi};U)$ for small values of U leads to a regular part and an irregular part:

$$\psi(\underline{x},\underline{\xi};U) = \psi_0(\underline{x},\underline{\xi}) + \tau\psi_1(\underline{x},\underline{\xi}) + \ldots + \tilde{\psi}_0(\underline{x},\underline{\xi}) + \nu^{-1}\tilde{\psi}_1(\underline{x},\underline{\xi}) + \ldots \tag{19}$$

where

$$\psi_0(\underline{x},\underline{\xi}) = 2g \int_{L_2} \frac{k\, e^{k(z+\zeta)}}{gk - \omega^2} J_0(kR)\,dk \tag{20}$$

$$\psi_1(\underline{x},\underline{\xi}) = 4ig^2\cos\theta' \int_{L_2} \frac{k^2 e^{k(z+\zeta)}}{(gk - w^2)^2} J_1(kR)\,dk \tag{21}$$

where $R^2 = (x-\xi)^2 + (y-\eta)^2$ and $\theta' = \operatorname{arctg}\left(\frac{y-\eta}{x-\xi}\right)$ and

$$\tilde{\psi}_0(\underline{x},\underline{\xi}) = -4\nu \int_0^{\pi/2} \exp[\nu(z+\zeta)\sec^2\theta]\sin[\nu(x-\xi)\sec\theta] * \cos[\nu(y-\eta)\sin\theta\sec^2\theta]\sec^2\theta\,d\theta \tag{22}$$

Expression (22) gives the interaction of the translating part of the Green's function with the oscillatory part. In the Appendix it is shown that due to the highly oscillatory behaviour the influence of (22) may be neglected in our first order correction for small values of τ.

4. EXPANSION OF THE SOURCE STRENGTH

In this section an approximate solution of (12) will be derived. Inserting (14) and (20) in (12) one obtains for like powers of τ the following set of equations:

$$-2\pi\sigma_0(\underline{x}) - \iint_S \sigma_0(\underline{\xi}) \frac{\partial G_0}{\partial n_x}(\underline{x},\underline{\xi})\, dS_\xi = 4\pi V_0(\underline{x}), \quad x \in S \tag{23}$$

and

$$\begin{aligned} -2\pi\sigma_1(\underline{x}) - \iint_S \sigma_1(\xi) \frac{\partial G_0}{\partial n_x}(\underline{x},\underline{\xi})\, dS_\xi &= -\iint_S \sigma_0(\underline{\xi}) \frac{\partial}{\partial n_x} \psi_1(\underline{x},\underline{\xi})\, dS_\xi + \\ &+ \frac{2i}{U} \iint_{FS} \frac{\partial G_0}{\partial n_x}(\underline{x},\underline{\xi}) \nabla\bar{\Phi}.\nabla\phi_0 dS_\xi + 4\pi V_1(\underline{x}) \end{aligned} \tag{24}$$

where $G_0(\underline{x},\underline{\xi}) = -\frac{1}{r} + \frac{1}{r_1} - \psi_0(\underline{x},\underline{\xi})$ is the zero speed pulsating wave source, and $V(\underline{x}) = V_0(\underline{x}) + \tau V_1(\underline{x}) + O(\tau^2)$.

This perturbation approach leads to a fast algorithm that takes into account speed effects once a fast method is available for the zero speed diffraction problem. At MARIN the diffraction program has been extended with the Finngreen subroutines of Newman. The diffraction program has been adjusted to compute the right-hand side of (24) as well.

The potential functions (14) now become:

$$\phi_0(\underline{x}) = -\frac{1}{4\pi} \iint_S \sigma_0(\underline{\xi}) G_0(\underline{x},\underline{\xi})\, dS_\xi$$

and

$$\begin{aligned} \phi_1(\underline{x}) = \frac{1}{4\pi} \iint_S \sigma_0(\underline{\xi}) \psi_1(\underline{x},\underline{\xi})\, dS_\xi - \frac{1}{4\pi} \iint_S \sigma_1(\underline{\xi}) G_0(\underline{x},\underline{\xi})\, dS_\xi + \\ + \frac{i}{2\pi U} \iint_{FS} G_0(\underline{x},\underline{\xi}) \nabla\bar{\Phi}.\nabla\phi_0 dS_\xi \end{aligned} \tag{25}$$

In the Appendix it is shown that the non-uniform term in the Green's function leads to contributions that are asymptotically small compared to the terms we have taken into account.

5. FIRST AND SECOND ORDER FORCES

Once the potentials $\phi_0(\underline{x})$ and $\phi_1(\underline{x})$ are known, the pressure is determined from Bernoulli's equation:

$$p(\underline{x},t) = gz - \rho\Phi_t - \frac{1}{2}\rho\nabla\Phi \cdot \nabla\Phi + p_0 + C(t) \tag{26}$$

The linearized pressure can be written as

$$p^{(1)}(\underline{x},t) = p_0(\underline{x},t) + \tau p_1(\underline{x},t) \tag{27}$$

with

$$\begin{aligned} p_0(\underline{x},t) &= -\rho\frac{\partial}{\partial t}\tilde{\phi}_0(\underline{x},t) \\ p_1(\underline{x},t) &= -\rho\frac{\partial}{\partial t}\tilde{\phi}_1(\underline{x},t) - \rho[U\frac{\partial}{\partial x}\tilde{\phi}_0(\underline{x},t) + \nabla\bar{\Phi}(\underline{x}).\nabla\tilde{\phi}_0(\underline{x},t)]/\tau \end{aligned} \tag{28}$$

Integration of the pressure over the mean wetted surface results in the hydrodynamic reaction forces in the usual coordinate system fixed to the ship

$$F_k = -\iint_S p \cdot n_k ds \tag{29}$$

Substitution of the pressure expansion (26, 27) gives:

$$\begin{aligned} F_k^{(0)} &= -\iint_S p_0 \cdot n_k ds \\ F_k^{(1)} &= -\iint_S p_1 \cdot n_k ds \end{aligned} \tag{30}$$

with

$$F_k = F_k^{(0)} + \tau F_k^{(1)}$$

For the unit motion in the j-mode one is now able to write the added mass and damping coefficients:

$$\begin{aligned} -\omega^2 a_{kj}^{(0)} &= \text{real } F_{kj}^{(0)} \\ -i\omega b_{kj}^{(0)} &= \text{imag } F_{kj}^{(0)} \end{aligned} \tag{31}$$

with similar definitions for $a_{kj}^{(1)}$ and $b_{kj}^{(1)}$.

F_{kj} is the reaction force in the k-mode due to a unit oscillation in the j-mode.

The second order (with respect to wave height) mean drift forces can be computed now. In the Bernoulli equation p and C(t) may be taken zero without loss of generality. Assuming that a point on the hull is carrying out a first order wave frequency motion $\underline{X}^{(1)}$ about a mean position $\underline{X}^{(0)}$ and applying a Taylor's expansion to the pressure in the mean position, the following expression is found:

$$p(\underline{x},t) = p^{(0)} + \varepsilon p^{(1)} + \varepsilon^2 p^{(2)} + O(\varepsilon^3) \qquad (32)$$

where is a measure of the wave height, $p^{(0)}$ the hydrostatic pressure and $p^{(1)}$ the first order pressure, given in (27).
The second order pressure is given by:

$$p^{(2)} = -\frac{1}{2}\rho|\nabla\tilde{\Phi}|^2 - \rho\tilde{\Phi}^{(2)} - \rho(\underline{X}^{(1)} \cdot \nabla\tilde{\Phi}_{1t}) \qquad (33)$$

After some algebraic manipulations, the final expression for the mean drift force becomes:

$$F^{(2)} = -g\int_{WL}\frac{1}{2}\rho|\zeta_r^{(1)}|^2\underline{n}\, d\, l + \alpha^{(1)} \times (M.\underline{\ddot{X}}_g^{(1)}) + \iint_{S_0}\frac{1}{2}\rho|\nabla\tilde{\Phi}|^2\underline{n}\, d\, S + \iint_{S_0}\rho(\underline{X}^{(1)} \cdot \nabla\tilde{\Phi}_t)\underline{n}\, d\, S \qquad (34)$$

This expression was derived by Pinkster [18]. We distinguish four contributions to the total mean drift force.

I First order relative wave elevation

$$-\frac{1}{2}\rho g\int_{WL}|\zeta_r^{(1)}|^2\underline{n}\, d\, l \qquad (35)$$

II Pressure drop due to first order velocity

$$\frac{1}{2}\rho\iint_{S_0}|\nabla\tilde{\Phi}|^2\underline{n}\, d\, S \qquad (36)$$

III Pressure due to products of first order pressure and first order motion

$$\rho \iint_{S_0} (\underline{X}^{(1)} \cdot \nabla \tilde{\Phi}_t) \underline{n} \, d \, S \tag{37}$$

IV Contribution due to products of first order angular motions and inertia forces

$$\underline{\alpha}^{(1)} \times (M . \ddot{\underline{X}}_g^{(1)}) \tag{38}$$

In exprsssion (34) the forward speed dependent potentials and derivatives of these potentials have to be evaluated at the mean waterline and the mean wetted surface.
The second order (with respect to the wave height) potential yields no contribution to the mean drift forces. It will be clear that we cannot make a perturbation series with respect to τ. The first order motion $\underline{X}^{(1)}$ depends on U in a complicated way, phase and amplitude are influenced by the small parameter τ. Therefore it is not possible to express (37) in a power series in τ uniformly. Similar arguments hold for some of the other terms. So, finally, the effect of the speed on the drift forces is computed by evaluating (34) numerically.

6. COMPUTATIONS OF LINEAR COEFFICIENTS

In order to evaluate the practicability of the porposed Green's function (20) and (21), calculations have been made on the one hand using an adapted version of the Finngreen algorithm and on the other hand using standard IMSL subroutines. We are able to transform the expression of ψ_1 in (21) into an expression which contains derivatives of ψ_0. Hence Finngreen can be used, for most of the terms, see Huijsmans and Hermans [9].
The computer time needed for the calculation of the forward speed influence was negligible compared with the zero speed computations. The total computer time increases by approximately 5%, for the computation of the added mass and damping coefficients.

At the moment very little data are available on the hydrodynamic reaction coefficients of ship type vessels at low Froude numbers.

In order to validate the present algorithm, computations have been made with a series 60 ship (block .70) as was used by Vugts [2]. Also computations have been performed by the Ecole Nationale Supérieure de Mécanique of Nantes [6] for the same ship and frequency range with a program developed by Grekas et al. [7]. Their approach is quite different.

As an example we show the results of added mass and damping coefficients, for the heave and pitch interaction.

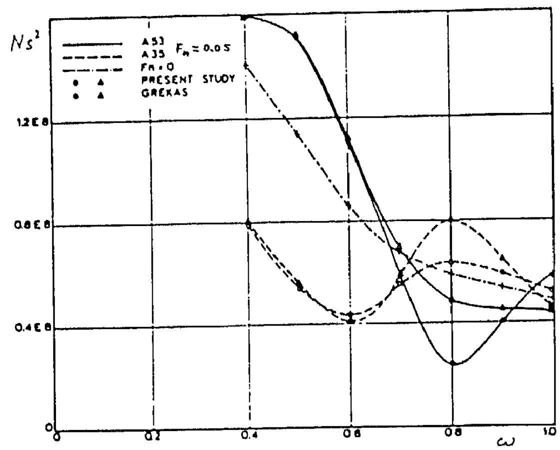

Fig. 3.

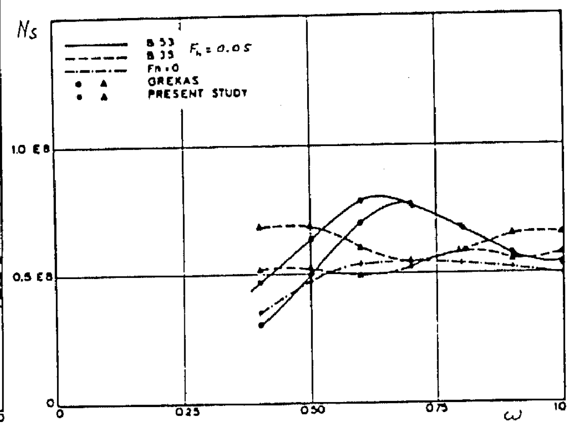

Fig. 4.

In spite of the reasonable correlation one still feels the need of the correlation with model test experiments.

7. COMPUTATIONS OF THE DRIFT FORCES

For the validation of the described numerical procedure to calculate the mean wavedrift forces, a number of model test experiments are analysed. The first order wave loads and added mass and damping coefficients were computed for a head sea condition for a tanker moored in deep water in a 4 knot current. The calculated response functions for the heave and pitch mode are presented in this figure. As shown the calculated heave and pitch motion response compare favourably with results of model test experiments.

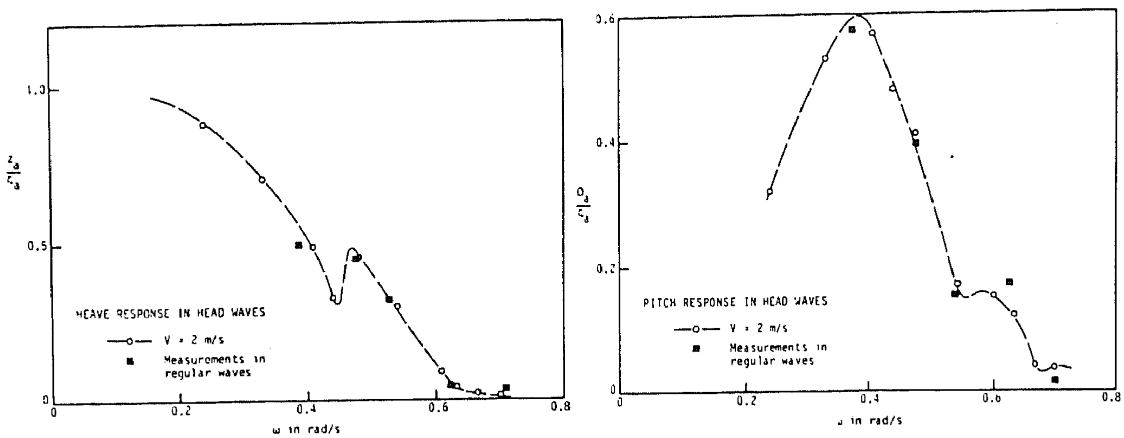

Fig. 5. Response of a 200 kDWT tanker in deep water

The four components to the drift force are computed separately. The contributions for the zero speed and 4 knot current speed are shown in the figures 6 and 7. The total mean drift force for the zero speed and the 4 knot current case are depicted in fig. 8. From the mean wave drift at zero speed and non-zero speed the so-called "wave damping" coefficient can be derived as shown by Wichers and Huijsmans [22]. For the underlying case the wave damping gives the following picture (fig. 9).

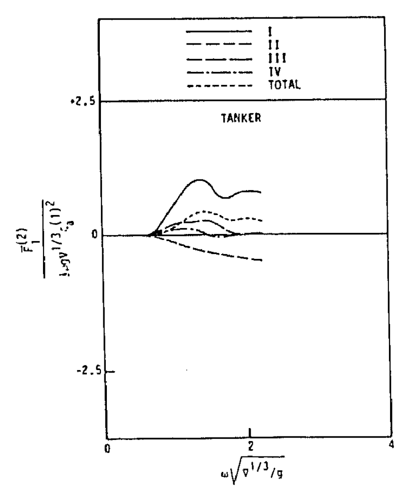

Fig. 6. Contributions to the mean wave drift force (zero speed)

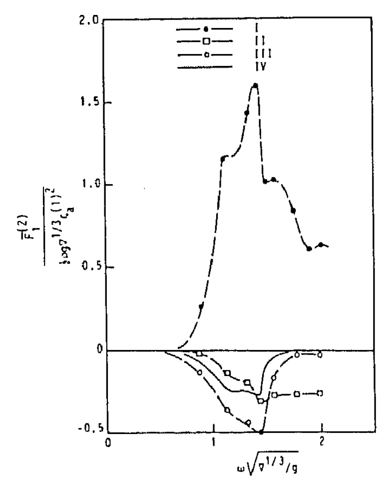

Fig. 7. Contributions to the mean wave drift force (4 knot)

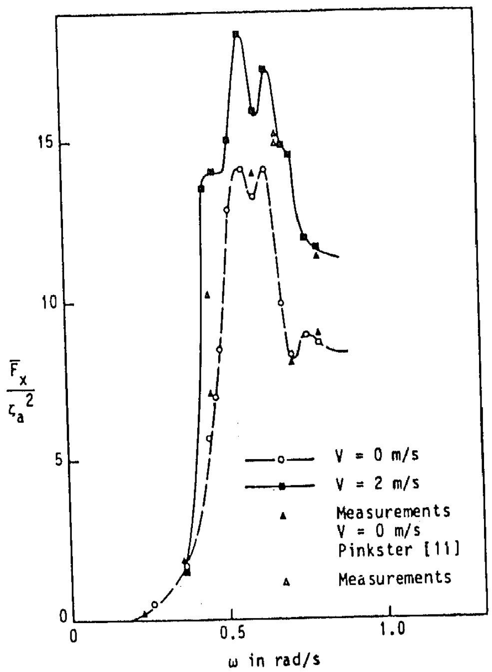

Fig. 8. Mean surge drift force in head waves for 200 kDWT tanker

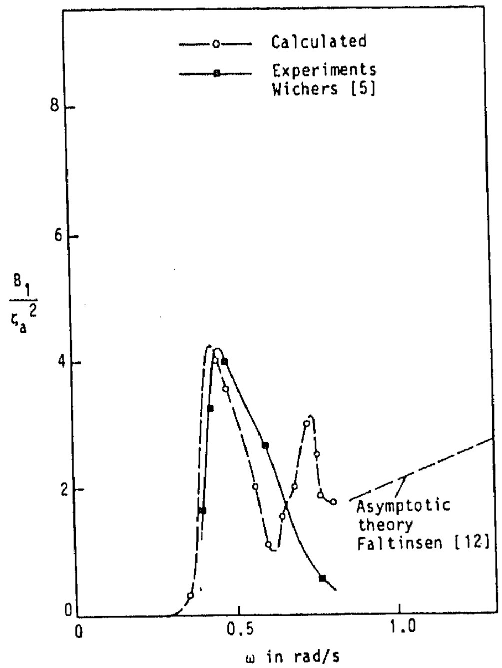

Fig. 9. "Wave damping" coefficient

In these examples the contribution of the free surface integrals are negligible. This is due to the fact that in the case of zero drift angle the double body stationary potential may be neglected. In the future computations will be carried out for ships moving at large drift angles. We expect that the influence of those terms becomes of importance.

APPENDIX

In this appendix we study the influence of $\tilde{\psi}_0(\underline{x},\underline{\xi})$ on the source strength and the potential. We rewrite the function $\tilde{\psi}_0$ as

$$\tilde{\psi}_0(\underline{x},\underline{\xi}) = \mathrm{Im}\,\chi\,(\underline{x},\underline{\xi}) = -\,\mathrm{Im}\,2\nu \int_{-\pi/2}^{\pi/2} \exp\,[\nu\sec^2\theta(z+\zeta+i\tilde{\omega})]\sec^2\theta d\theta \qquad \text{(A.1)}$$

with $\tilde{\omega} = (x-\xi)\cos\theta - (y-\eta)\sin\theta$ and $\nu = \frac{g}{U^2}$.

First it will be shown that the integral

$$I(x,z) = \iint_S \sigma_0(\underline{\xi}) \frac{\partial\tilde{\psi}_0}{\partial n_x}(\underline{x},\underline{\xi})dS_\xi \qquad \text{(A.2)}$$

is asymptotically small. In the case we are dealing with a thin ship the analysis can be carried out without a reconsideration of the stationary potential. In the general case a uniformly valid stationary potential is required to complete the proof. We will indicate where this has to be done. For the final analysis a similar approach as in the low Froude number wave resistance problem is appropriate. In the forthcoming Ph.D. thesis of Brandsma the wave resistance problem is solved for small values of the Froude number. It is reasonable to assume in our case that a similar analysis can be carried out. The result will be that contributions to (A.2) come from the end points of the ship. The results have similar, or smaller, asymptotic behaviour as in the thin ship case. Hence, we continue with the thin ship case.

For the thin ship the source distribution (A.2) can be written as a distribution along the projection $\tilde{S}$ of S on the (x,z) plane, moreover, the approximation $\partial/\partial n_x = \partial/\partial y$ holds. We then obtain on S:

$$\frac{\partial \chi}{\partial y} = 2\nu^2 i \int_{-\pi/2}^{\pi/2} \sec^4\theta \, \sin\theta \, \exp[\nu\sec^2\theta \, (z + \zeta + i(x-\xi)\cos\theta))]d\theta \qquad (A.3)$$

We therefore consider the integral

$$I(x,z) = 2\nu^2 i \int_{-\pi/2}^{\pi/2} \{ \iint_{S} \sigma_0(\xi,\zeta) \exp\,[\nu\sec^2\theta(z+\zeta) + i\nu\sec\theta(x-\xi)]d\xi d\zeta\} \sec^4\theta \sin\theta d\theta \qquad (A.4)$$

in the limit $\nu = g/U^2 \to \infty$.

First, we consider the integral along $\tilde{S}$. Integration by parts results in the major contribution because no stationary points are situated on $\tilde{S}$ or its boundary. We obtain:

$$I^{\pm}(x,z) = \mp 2 \int_{-\pi/2}^{\pi/2} \sec\theta \sin\theta \sigma_0(\xi^{\pm},0) \exp[\nu(z\sec^2\theta + i(x-\xi^{\pm})\sec\theta)]d\theta + \qquad (A.5)$$

$$+ O(\frac{1}{\nu}) \text{ as } \nu \to \infty$$

Here $(\xi^{\pm},0)$ are the endpoints of the ship at the waterline. The main contribution is generated by the endpoints. This is well known for the wave resistance problem. Further asymptotic evaluation by means of partial integration can be carried out because no stationary points show up.

In the general case we find stationary points. However, they are generated by the nonuniformity of the stationary potential. They must be disregarded. In this case we find:

$$I^{\pm}(x,z) = 0(\frac{1}{\nu^2}) \quad \text{as} \quad \nu \to \infty \tag{A.6}$$

Actually, it can be proven that the contribution of $\tilde{\psi}_0$ to $I(x,z)$ is asymptotically zero.

To obtain insight in the influence of $\tilde{\psi}_0(\underline{x},\underline{\xi})$ on the potential function we study the integral:

$$J(x) = -2\nu \int_{-\pi/2}^{\pi/2} \{\iint_{S} \sigma_0(\xi,\zeta) \exp\, [\nu \sec^2\theta \; (z + \zeta + i((x-\xi)\cos\theta - y\sin\theta))] d\xi d\zeta\} . \sec^2\theta d\theta \tag{A.7}$$

The integral along $\tilde{S}$ can be evaluated by means of partial integration. We obtain:

$$J^{\pm}(x) = \pm \frac{2}{i\nu}\, \sigma_0(\xi^{\pm},0) \int_{-\pi/2}^{\pi/2} \exp\, [\nu(z\sec^2\theta + i(x-\xi^{\pm})\sec\theta - iy\sec^2\theta\sin\theta)] . \frac{d\theta}{\sec^3\theta} \tag{A.8}$$

For arbitrary values of x a further asymptotic expansion of $J^{\pm}(x)$ is possible by means of the method of stationary phase. A Kelvin pattern is generated at the bow and the stern. The main term behaves like:

$$J^{\pm}(x) = 0\left[\frac{1}{\nu^{3/2}}\right] = 0(U^3) \tag{A.9}$$

which means that a contribution at higher order than the linear term has been obtained in the wave height. For the calculation of the pressure at the ship a similar analysis shows that higher order terms are obtained as well. Hence, we may neglect these terms if one is interested in linear correction terms.

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